Balancing Decision-making Errors when Testing Hypotheses with the Binomial Test

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Steven G. Saiz, Environmental Scientist
Division of Water Quality
State Water Resources Control Board
California Environmental Protection

Summary

Section 303(d) of the Clean Water Act requires states to establish a list of water bodies that do not meet water quality standards. State Water Resources Control Board staff recently proposed the binomial hypothesis test when deciding to list or delist a water body. The traditional binomial test effectively controls the alpha error rate (i.e., the chance of incorrectly rejecting a true null hypothesis) at or below the proposed nominal significance level of 10%. Several authors, however, have suggested that the beta error rate (i.e., the chance of incorrectly failing to reject a false null hypothesis) should be considered when listing or delisting and that alpha and beta rates should be equally balanced, if possible. The methodology and probability equations used to derive sampling plans based on observed exceedances is reviewed, both for the proposed traditional binomial test and a binomial test based on a balanced error approach. Approximate error balancing provides an equitable way to decide whether a water body should be listed or delisted, as long as a sufficient number of samples are collected to keep the error rates below a moderate level.

Introduction

Section 303(d) of the Clean Water Act requires states to establish a list of water bodies that do not meet water quality standards. Regulatory decisions to list water bodies or to "delist" water bodies (i.e., remove the water body from the 303(d) list) can either be correct or incorrect decisions. Although states desire to always make correct decisions when listing or delisting, this is not always possible. Because of this, statistical hypothesis testing techniques such as the binomial test are used to keep one type of decision-making error, the alpha error, at an acceptably low level. This increases confidence when making regulatory decisions.

The alpha, α, statistical error rate is also known as the Type I error rate and is defined as the probability of incorrectly rejecting a true null hypothesis. Similarly, the beta, β, statistical error rate is also known as the Type II error rate and is defined as the probability of incorrectly failing to reject a false null hypothesis. Only one or neither of these errors can be made for a given listing assessment.

In contrast to the usual statistical approach of controlling only the alpha error rate, Smith et al. (2001) discussed the idea of making alpha and beta error rates equal for each given sample size. An example was presented showing that alpha and beta error rates can be kept below 20% with sample sizes of around 25. Balancing of both decision-making error rates was also addressed by the United States Environmental Protection Agency in their Consolidated Assessment and Listing Methodology Guidance (CALM, Appendix D) written by Riggs and Aragon (2002). This guidance recommends that tests which balance both alpha and beta errors at levels below 15% are preferable to tests designed only to minimize alpha errors.

This SWRCB staff report will explain and comment on the balanced error approach and compare balanced error sampling plans with a proposed SWRCB listing Policy (SWRCB 2003).

Binomial Hypothesis Testing with a Fixed Significance Level

SWRCB (2003) staff proposed the use of the traditional binomial hypothesis test with a significance level of 10% when deciding to list or delist. All hypothesis tests initially require setting a fixed, nominal significance level. The significance level is traditionally set at 1%, 5% or 10%, "but there is no reason why other values should not be used" (Helsel and Hirsch 2002, p.106). If, for example, the testing is conducted as a first cut to separate sites into "high" or "low" contamination areas, the significance level might be set to 10% or 20%.

The binomial test is identical to acceptance sampling by *attributes* (Gibra 1973): random samples are evaluated to be either above or below the applicable water quality standard. A water body is listed if the number of exceedances k in N samples equals or exceeds a critical value *klist*. Likewise, a water body is delisted if $k \leq kdelist$ in a sample of N. This process is called a *single acceptance sampling plan* since the decision is based on a single sample of size N (Gibra 1973).

klist and *kdelist* are determined iteratively as the smallest and largest number of exceedances, respectively, such that α is less than or equal to the desired nominal significance level, given N and a tolerable regulatory exceedance rate threshold r_1. Note that β is not used in the determination of either *klist* or *kdelist*.

The following procedures for listing and delisting are based on the currently proposed approach (SWRCB 2003) which controls α but not β.

Procedure for Listing with a Fixed Significance Level

A standard null hypothesis, H_o is used for listing a water body. The default assumption is that the true, but unknown, exceedance rate, r, is less than or equal to the regulatory exceedance rate, r_1. The tested one-sided hypotheses are the null hypothesis, H_o: $r \leq r_1$, versus the alternate hypothesis, H_a: $r > r_1$.

To find *klist*, let *klist* = 0 initially. Then calculate α from the right tail probability of the cumulative binomial distribution:

$$\alpha = P(k \geq klist \mid r_1, N) = \sum_{k=klist}^{N} \left(\frac{N!}{k!(N-k)!} \right) r_1^{\ k} (1-r_1)^{(N-k)}$$

$$= \mathrm{I}(r_1, klist, N - klist + 1)$$

$$= \text{BINOMDIST}(N\text{-}klist,\ N,\ 1\text{-}r_1,\ \text{TRUE}) \qquad \text{(Equation 1)}$$

where $\mathrm{I}(x,a,b)$ is the incomplete beta function (Abramowitz and Stegun 1972) and BINOMDIST( ) is an Excel software function that returns cumulative left tail binomial probabilities. If α is greater than the desired nominal significance level then add one to *klist* and repeat until α is less than or equal to the desired nominal significance level. Consequently, *klist* is a function of three input values: N, r_1, and the desired nominal significance level.

Under the null hypothesis, the expected number (i.e., the average value) of exceedances is the product r_1N. If observed exceedances k equals or exceeds *klist*, the null hypothesis is rejected. The logical outcome of rejecting the null hypothesis is that the water body is not meeting water quality standards should be placed on the 303(d) list.

As an example, consider a situation where 25 samples are randomly collected and the binomial test is applied at the 0.10 nominal significant level. To find *klist* under the null hypothesis of H_o: $r \leq r_1 = 0.1$, refer to a table of cumulative binomial probabilities (Table 1). In this example, we expect to see 2 or 3 exceedances on average, but 5 or more exceedances is sufficient evidence to list the water body with (1-α)100% = (1-0.0980)100% = 90.2% confidence. Another way to express this is to say that when the null hypothesis is true we will list a water body having 5 or more exceedances 90.2% of the time, on average.

Procedure for Delisting with a Fixed Significance Level

A "reversed" null hypothesis is used for delisting a water body. The default assumption is that the true, but unknown, exceedance rate, r, is greater than or equal to the regulatory exceedance rate, H_o: $r \geq r_1$, versus the alternate hypothesis, H_a: $r < r_1$.

To find *kdelist*, let *kdelist* = 0 initially. Then calculate α from the left tail probability of the cumulative binomial distribution:

$$\alpha = P(k \le kdelist \mid r_1, N) = \sum_{k=0}^{kdelist} \left(\frac{N!}{k!(N-k)!} \right) r_1^{\ k} (1-r_1)^{(N-k)}$$

$$= 1 - \sum_{k=kdelist+1}^{N} \left(\frac{N!}{k!(N-k)!} \right) r_1^{\ k} (1-r_1)^{(N-k)}$$

$$= 1 - \mathrm{I}(r_1, kdelist+1, N-(kdelist+1)+1) = 1 - \mathrm{I}(r_1, kdelist+1, N-kdelist)$$

$$= \text{BINOMDIST}(kdelist, N, r_1, \text{TRUE}) \qquad \text{(Equation 2)}$$

If α is less than the desired nominal significance level then add one to *klist* and repeat until α is less than or equal to the desired nominal significance level. The null hypothesis is rejected if $k \le kdelist$, and the water body is considered to meet water quality standards.

Note that for delisting with small sample sizes, α may be larger than the desired significance level even when *kdelist* = 0. The minimum sample size required for delisting is equivalent to the sample size required for an upper one-sided non-parametric tolerance limit (Owen 1962),

$$N = \frac{\ln(\alpha)}{\ln(1-r_1)}.$$

In practice, *N* is rounded up to the nearest integer. For example, using a nominal significance level of 0.1 and a regulatory exceedance rate of 0.1 the minimum sample size required is ln(0.1)/ln(1-0.1) = 21.9. Rounded up, a minimum of 22 sample would be required for delisting.

Consider again the previous example with *N* = 25. Since there are more than 22 samples, *kdelist* can be determined. Referring to the binomial probabilities in Table 1, under the reverse null hypothesis of H_o: $r \ge r_1 = 0.1$, *kdelist* = 0. This indicates that zero exceedances in a sample of 25 would be sufficient evidence to delist the water body with (1-α)100% = (1-0.0718)100% = 92.8% confidence.

<u>The Draft SWRCB Policy Sampling Plan</u>

Table 2 lists the critical number of exceedances required to list a water body and to delist a water body as a function of sample size as proposed in the draft SWRCB Policy. These critical observed exceedances were calculated using the above procedures with a nominal significance level of 0.10.

Calculating Alpha and Beta Error Rates

Decision-making error rates associated with the traditional binomial test can be determined analytically from the cumulative binomial probability distribution. The binomial test effectively controls α, but not β. A graph showing the theoretical probability of rejecting the null hypothesis on the vertical axis versus *r* on the horizontal axis is known as a power curve. The mathematical complement of a power curve is an operating characteristic curve (OC) curve. An OC curve is a power curve flipped along the horizontal axis by subtracting the power curve probability from unity.

Procedure for Listing

For listing water bodies, the probability of rejecting the standard null hypothesis is calculated using the right tail probability of the cumulative binomial distribution and selected values of *r* (i.e., alternate exceedance rates) within the interval [0,1]:

$$P(reject\ H_0) = P(k \geq klist \mid klist, N)$$

$$= \sum_{k=klist}^{N} \left(\frac{N!}{k!(N-k)!} \right) r^{k} (1-r)^{(N-k)}$$

$$= \mathrm{I}(r, klist, N-klist+1)$$

= BINOMDIST(*N-klist*, *N*, 1-*r*, TRUE) (Equation 3)

This probability equals α when the null hypothesis is true and power (1 - β) when the null hypothesis is false. Under the standard hypothesis, α is the probability of incorrectly listing a clean water body while β is the probability of incorrectly failing to list a contaminated water body.

The probability of *not rejecting* the standard null hypothesis is the complement of Equation 3:

$$P(not\ reject\ H_0) = 1 - P(reject\ H_0) = P(k \leq klist - 1 \mid klist, N)$$

$$= \sum_{k=0}^{klist-1} \left(\frac{N!}{k!(N-k)!} \right) r^{k} (1-r)^{(N-k)}$$

$$= 1 - \mathrm{I}(r, klist, N-klist+1)$$

= BINOMDIST(*klist-1*, *N*, *r*, TRUE) (Equation 4)

This probability equals the confidence coefficient (1-α) when the null hypothesis is true and β when the null hypothesis is false.

Using the example of $N = 25$, Figure 1 illustrates these probabilities as a function of alternate exceedance rates for the standard null hypothesis. This graph simultaneously depicts alpha or power (via Equation 3) and confidence or beta (via Equation 4).

Procedure for Delisting

For delisting water bodies, the probability of rejecting the reverse null hypothesis is calculated using the left tail probability of the cumulative binomial distribution and selected values of *r* within the interval [0,1]:

$$P(reject\ H_0) = P(k \le kdelist \mid kdelist, N)$$

$$= \sum_{k=0}^{kdelist} \left(\frac{N!}{k!(N-k)!} \right) r^{k} (1-r)^{(N-k)}$$

$$= 1 - \mathrm{I}(r, kdelist + 1, N - kdelist)$$

$$= \text{BINOMDIST}(kdelist, N, r, \text{TRUE}) \qquad \text{(Equation 5)}$$

Again, this probability equals α when the null hypothesis is true and power (i.e., 1 - β) when the null hypothesis is false. However, under the reverse hypothesis **the nature of the errors are reversed**: α is now the probability of incorrectly failing to list (delisting) a water body that doesn't meet standards while β is the probability of incorrectly listing (not delisting) a water body that does meet standards.

The probability of *not rejecting* the reverse null hypothesis is the complement of Equation 5:

$$P(not\ reject\ H_0) = 1 - P(reject\ H_0) = P(k \ge kdelist + 1 \mid kdelist, N)$$

$$= \sum_{k=kdelist+1}^{N} \left(\frac{N!}{k!(N-k)!} \right) r^{k} (1-r)^{(N-k)}$$

$$= \mathrm{I}(r, kdelist + 1, N - kdelist)$$

$$= \text{BINOMDIST}(N\text{-}kdelist\text{-}1, N, 1\text{-}r, \text{TRUE}) \qquad \text{(Equation 6)}$$

This probability equals the confidence coefficient (1-α) when the null hypothesis is true and β when the null hypothesis is false.

Again, using the example of $N = 25$, Figure 2 illustrates these probabilities as a function of alternate exceedance rates for the standard null hypothesis.

Error Rates in the Draft SWRCB Policy

Figures 1 and 2 show that β decreases rapidly as the true exceedance rate moves away from the hypothesized exceedance rate. In other words, the chance β of incorrectly rejecting the null hypothesis increases as the true exceedance rate gets *closer* to the hypothesized exceedance rate. The largest decision-making errors, therefore, are incurred when the difference between the hypothesized condition and the true condition is very small (i.e., when the *effect size* is very small). This can be contrasted with actual environmental effects, which continually worsen as the true exceedance rate increases toward 100%.

Figure 3 shows maximal statistical error rates associated with the draft SWRCB Policy sampling plans in Table 2 for sample sizes up to 120. Notice that α is controlled at levels less than or equal to 0.10 for all sample sizes shown. The β error rate, however, is consistently greater than 0.90. In addition, larger sample sizes do not appreciably lower maximal β rates. Rates for β of 0.2 or less are generally desirable but are not achieved using this conventional hypothesis testing approach.

The top graph of Figure 3 emphasizes that when deciding not to list a water body (i.e., accepting the null hypothesis of H_o: $r \leq 0.1$) we have a high probability ($\beta > 0.90$) of "missing" a water body that should, in fact, be listed. This decision error is greatest when the true alternate exceedance rate is very close to, but greater than, the hypothesized exceedance rate of $r = 0.10$.

In contrast, the lower graph of Figure 3 emphasizes that when deciding to keep water body on the 303(d) list (i.e., accepting the null hypotheses of H_o: $r \geq 0.1$) we have a high probability ($\beta > 0.90$) of incorrectly failing to remove a water body from the 303(d) list. Again, this decision error is greatest when the true exceedance rate is very close to, but less than, the hypothesized exceedance rate of $r = 0.10$.

Balancing Alpha and Beta Errors

The binomial test, like most statistical hypothesis testing procedures, will control the maximum α rate at a value below the nominal significance level for most sample sizes. In contrast, the magnitude of β depends on several factors, including α, the population variance, the effect size, and sample size. Generally, α varies inversely with β, and control of β is traditionally sought through the appropriate selection of sample size (Gibra 1973, p.208) or through the use of a more powerful statistical test (Helsel and Hirsch 2002, p.107).

Alternatives to controlling only the α rate are possible. Mapstone (1995) argued against adhering to a fixed and arbitrary α, advocating instead for the consideration of economic, environmental, social, and political consequences of both α and β decision-making errors. In the absence of further information, Mapstone recommended that decision errors should be weighted equally, i.e., $\alpha = \beta$. In addition, he recommended that decision-makers define a level of impact essential to detect – an effect size. Furthermore, Mapstone suggested that the effect size is perhaps the most critical aspect of environmental impact decision-making and is a biological (or chemical, physical, aesthetic, economic, etc.) decision, not simply a statistical decision.

The effect size is variously called the *grey region* within the Data Quality Objectives (DQO) process (Millard and Neerchal 2001, p. 22) or the *indifferent zone* (Gibra 1973, p. 493) within the acceptance sampling process. For Clean Water Act 303(d) listing and delisting, the effect size represents the range of true exceedance rates where the consequences of decision errors are relatively minor.

Riggs and Aragon (2002, Sec. D.5) applied the error balancing approach of Smith (2001) to the 303(d) listing process. To balance errors, *klist* and *kdelist* are determined in a manner different than previously described.

Balanced Error Approach for Listing

Figure 4 is a magnification of the lower portion of Figure 1. Examination of Figure 4 reveals that an alternate exceedance rate value r_2 exists such that $\alpha = \beta$. This can be envisioned as a horizontal line passing through the α curve and the β curve with vertical lines indicating r_1 and r_2. In fact, an infinite number of alternate exceedance rate pairs (r_1, r_2) exist that will balance α and β at a varying levels for a given N and *klist*. As the balanced error level decreases the effect size ($r_2 - r_1$) increases since r_1 must decrease and r_2 must increase. Holding r_1 or r_2 constant will affect the magnitude of α and β and the degree to which these errors can be balanced.

The approach taken by Riggs and Aragon (2002) for listing is to first define N, r_1, and r_2. Next, *klist* is determined iteratively as the k value that minimizes the absolute difference between α and β. The minimized quantity $|\alpha - \beta|$ can be expressed using Equation 3 for α and Equation 4 for β:

$$|\alpha - \beta| = |\ \mathrm{I}(r_1, klist, N - klist + 1) - [1 - \mathrm{I}(r_2, klist, N - klist + 1)]\ | \quad \text{(Equation 7)}$$

where $r_1 < r_2 < 1$. An equivalent procedure is to first define N, r_1, and the effect size ($r_2 - r_1$).

This minimization algorithm is analogous to the minimum squared deviation technique used in statistical curve-fitting of data. Errors will balance perfectly when the minimized quantity is zero. However, because of the discrete nature of the binomial probability distribution only approximate balancing of α and β is possible, especially with smaller sample sizes.

Figure 5 illustrates the determination of *klist* using the above balanced error approach when $N = 25$, $r_1 = 0.1$, and $r_2 = 0.25$, giving an effect size of 0.15. In this example, five observed exceedances gives the minimim absolute error difference, but the errors still cannot be balanced equitably since β is over two times larger than α.

Balanced Error Approach for Delisting

For delisting, the Riggs and Aragon (2002) approach is to again define N, r_1, and r_2, but this time r_2 is a value less than r_1. *kdelist* is determined as the k value that minimizes the absolute difference between α and β. The minimized quantity $|\alpha - \beta|$ can be expressed using Equation 5 for α and Equation 6 for β:

$$|\alpha - \beta| = |\, [1 - \mathrm{I}(r_1, kdelist+1, N-kdelist)] - \mathrm{I}(r_2, kdelist+1, N-kdelist)\, | \quad \text{(Equation 8)}$$

where $r_2 < r_1 < 1$.

SWRCB staff developed a computer program, BinomBal.exe, that will evaluate the minimized quantities in Equations 7 or 9 to derive *klist* or *kdelist*.

Choosing Appropriate Starting Values with the Balanced Error Approach

An important consideration when calculating *klist* and *kdelist* by the balanced error approach is the values assigned to r_1 and r_2 for both listing and delisting. It is possible, and undesirable, to assign r_1 and r_2 values that would result in conflicting decision rules for listing and delisting. Under such starting values, a set of observed exceedances will exist that simultaneously result in a decision to list under the standard null hypothesis and a decision to delist under the reverse null hypothesis for a given N.

For example, given $N = 25$ and for listing $r_1 = 0.10$ and $r_2 = 0.25$, but for delisting $r_1 = 0.40$ and $r_2 = 0.25$. Using the balanced error approach leads to *klist* = 5 or more exceedances and *kdelist* = 6 or less exceedances. A water body listed with 5 or 6 exceedances in a sample of 25 could then immediately be delisted! Generally, the balanced error approach should result in a *kdelist* value that is at least one exceedance less than *klist*.

A special case exists when r_1 (listing) = r_2 (delisting) and r_2 (listing) = r_1 (delisting). This special case of r_1 and r_2 starting values results in the equality of the minimized error quantities in Equations 7 and 8. Equating these equations means that *kdelist* will always be one less than *klist*. Thus, α for listing becomes exactly equal to β for delisting and vise-versa. This reversal and equality of errors for listing and delisting is desirable because conflicting decisions based on which null hypothesis is chosen (standard versus reversed) will then be eliminated. Indeed, Smith (2001) noted that balanced decision error rates are less affected by switching the null and alternative hypothesis.

Comparison of the Draft SWRCB Policy with the Balanced Error Approach

The balanced error approach is useful because it considers both types of decision-making errors, α and β, rather than only α when designing sampling plans. Another objective is to maintain these balanced error rates at or below an acceptable magnitude. Although Riggs and Aragon (2002) suggested that a moderate acceptable magnitude for balancing errors is 15%, the choice of values for α and β rates is a subjective policy decision (Millard and Neerchal 2001). Nevertheless, a pre-defined maximum acceptable error for both α and β will allow the determination of acceptable sample sizes to use for listing and delisting.

Tables 3-5 list selected sampling plans and the critical number of exceedances required to list or delist a water body as a function of sample size when applying the balanced error approach with no conflicting decision rules. More detailed output from the BinomBal program for these selected sampling plans is included in the Appendix. Figures 6-8 display statistical error rates associated with the sampling plans of Tables 3-5. Notice that by using the balanced error approach both α and β decrease appreciably with increasing *N*. Lowered α and β rates using the balanced error approach contrast sharply with the higher β error rates expected when using the traditional binomial test.

Figure 9 directly compares the selected balanced error sampling plans with the existing SWRCB Policy sampling plans. With small sample sizes under 60 the balanced error plans require fewer exceedances to list a water body and allow more exceedances when delisting a water body.

Appropriate sample sizes required to achieve desired error rates can be read from Figures 6-8. If the effect size is 15% (Figures 6 and 8) and we wish to maintain both α and β rates at or below 0.15 then about 30 samples are needed. To maintain both α and β rates at or below 0.20 about 20 samples are needed.

A 10% effect size (Figure 7) results in more rigorous sample size requirements. To maintain both α and β rates at or below 0.15 about 65 samples are needed. To maintain both α and β rates at or below 0.20 about 50 samples are needed.

In conclusion, the error balancing approach is an equitable way to decide whether a water body should be listed or delisted – as long as a sufficient number of samples are collected to keep the error rates below at moderate levels of 15-20%.

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Table 1. Binomial probability table for $N = 25$ when the true exceedance rate, r_1, is 0.10. The expected number of exceedances is $r_1N = 2.5$. Using a nominal significance level of 0.10 and the column of right tail probabilities, *klist* = 5 and the exact significance level is 0.0980. Similarly, using left tail probabilities, *kdelist* = 0 and the exact significance level is 0.0718

Number of Exceedances, *k*	Probability of Exactly *k* Exceedances	Left Tail Cumulative Probability of *k or less* Exceedances	Right Tail Cumulative Probability of *k or more* Exceedances
0	0.0718	0.0718	1.0000
1	0.1994	0.2712	0.9282
2	0.2659	0.5371	0.7288
3	0.2265	0.7636	0.4629
4	0.1384	0.9020	0.2364
5	0.0646	0.9666	0.0980
6	0.0239	0.9905	0.0334
7	0.0072	0.9977	0.0095
8	0.0018	0.9995	0.0023
9	0.0004	0.9999	0.0005
10	0.0001	1.0000	0.0001
11	0.0000	1.0000	0.0000
12	0.0000	1.0000	0.0000
13	0.0000	1.0000	0.0000
14	0.0000	1.0000	0.0000
15	0.0000	1.0000	0.0000
16	0.0000	1.0000	0.0000
17	0.0000	1.0000	0.0000
18	0.0000	1.0000	0.0000
19	0.0000	1.0000	0.0000
20	0.0000	1.0000	0.0000
21	0.0000	1.0000	0.0000
22	0.0000	1.0000	0.0000
23	0.0000	1.0000	0.0000
24	0.0000	1.0000	0.0000
25	0.0000	1.0000	0.0000

Table 2. Observed exceedances required to reject the null hypothesis as presented in the SWRCB draft Policy (December 2, 2003 Version). Nominal significance level is 0.10*.

To List with 90% Confidence H_o: $r \leq 0.10$ H_a: $r > 0.10$			To Delist with 90% Confidence H_o: $r \geq 0.10$ H_a: $r < 0.10$	
Sample Size, *N*	List if this number or greater, *klist*		Sample Size, *N*	Delist if this number or fewer, *kdelist*
10 - 11	3		22 – 37	0
12 – 18	4		38 – 51	1
19 - 25	5		52 – 64	2
26 – 32	6		65 – 77	3
33 – 40	7		78 – 90	4
41 – 47	8		91 – 103	5
48 – 55	9		104 – 115	6
56 – 63	10		116 – 120	7
64 – 71	11			
72 – 79	12			
80 – 88	13			
89 – 96	14			
97 – 104	15			
105 – 113	16			
114 – 120	17			

*$\alpha \leq 0.1$, β not controlled.

Table 3. Observed exceedances required to reject the null hypothesis based on the balanced error approach. Effect size = 15%.

List Sample Plan H_o: $r \leq 0.05$ H_a: $r > 0.20$			*Delist Sample Plan* H_o: $r \geq 0.20$ H_a: $r < 0.05$	
Sample Size, *N*	List if this number or greater, *klist*		Sample Size, *N*	Delist if this number or fewer, *kdelist*
1 – 9	1		1 – 9	0
10 – 19	2		10 – 19	1
20 (21)*– 28	3		20 (21)*– 28	2
29** – 37	4		29** – 37	3
38 – 46	5		38 – 46	4
47 – 55	6		47 – 55	5
56 – 64	7		56 – 64	6
65 – 73	8		65 – 73	7
74 – 82	9		74 – 82	8
83 – 91	10		83 – 91	9
92 – 100	11		92 – 100	10
101 – 109	12		101 – 109	11
110 – 118	13		110 – 118	12
119 – 120	14		119 – 120	13

* α and $\beta < 0.2$ at Sample Size = 21.
** α and $\beta < 0.15$ at Sample Size = 29

Table 4. Observed exceedances required to reject the null hypothesis based on the balanced error approach. Effect size = 10%. Compare delisting values with Table 5 of Riggs and Aragon (2002).

List Sample Plan H_o: $r \leq 0.10$ H_a: $r > 0.20$			*Delist Sample Plan* H_o: $r \leq 0.20$ H_a: $r < 0.10$	
Sample Size, *N*	List if this number or greater, *klist*		Sample Size, *N*	Delist if this number or fewer, *kdelist*
1 – 7	1		1 – 7	0
8 – 14	2		8 – 14	1
15 – 21	3		15 – 21	2
22 – 28	4		22 – 28	3
29 – 35	5		29 – 35	4
36 – 42	6		36 – 42	5
43 – 48	7		43 – 48	6
49 (50)*– 55	8		49 (50)*– 55	7
56 – 62	9		56 – 62	8
63 (65)** – 69	10		63 (65)** – 69	9
70 – 76	11		70 – 76	10
77 – 83	12		77 – 83	11
84 – 90	13		84 – 90	12
91 – 97	14		91 – 97	13
98 – 104	15		98 – 104	14
105 – 110	16		105 – 110	15
111 – 117	17		111 – 117	16
118 – 120	18		118 – 120	17

* α and $\beta < 0.2$ at Sample Size = 50.
** α and $\beta < 0.15$ at Sample Size = 65

Table 5. Observed exceedances required to reject the null hypothesis based on the balanced error approach. Effect size = 15%. Compare listing values with Table 4 of Riggs and Aragon (2002).

List Sample Plan H_o: $r \leq 0.10$ H_a: $r > 0.25$			*Delist Sample Plan* H_o: $r \geq 0.25$ H_a: $r < 0.10$	
Sample Size, *N*	List if this number or greater, *klist*		Sample Size, *N*	Delist if this number or fewer, *kdelist*
1 – 6	1		1 – 6	0
7 – 12	2		7 – 12	1
13 – 18	3		13 – 18	2
19 – 24	4		19 – 24	3
25 (26)*– 30	5		25 (26)*– 30	4
31 – 36	6		31 – 36	5
37 – 42	7		37 – 42	6
43 – 48	8		43 – 48	7
49 – 54	9		49 – 54	8
55 – 60	10		55 – 60	9
61 – 66	11		61 – 66	10
67 – 72	12		67 – 72	11
73 – 78	13		73 – 78	12
79 – 84	14		79 – 84	13
85 – 91	15		85 – 91	14
92 – 97	16		92 – 97	15
98 – 103	17		98 – 103	16
104 – 109	18		104 – 109	17
110 – 115	19		110 – 115	18
116 – 120	20		116 – 120	19

*α and $\beta < 0.2$ at Sample Size = 26.
** α and $\beta < 0.15$ at Sample Size = 33

Figure 1. Probabilities of rejecting (solid red line) and not rejecting (dashed blue line) the standard null hypothesis H_o: $r \leq r_1 = 0.1$ when using the binomial model. Alpha error is the solid red line to the left of the vertical dashed line; power is the red line to the right. Beta error is the solid blue line to the right of the vertical dashed line; confidence is the blue line to the left.

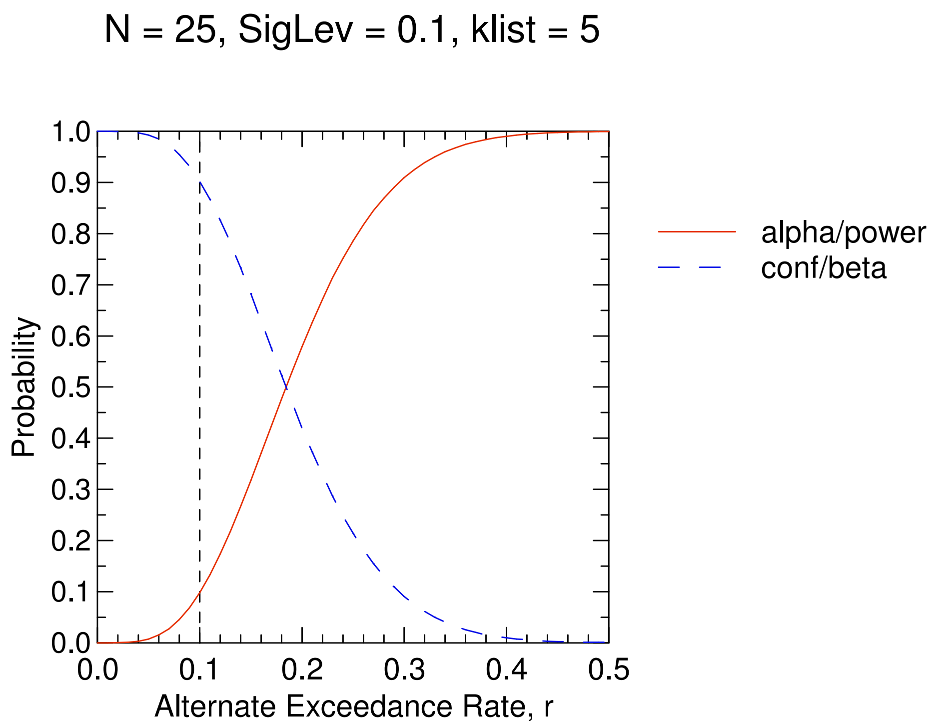


Figure 2. Probabilities of rejecting (solid red line) and not rejecting (blue line) the reverse null hypothesis H_o: $r \geq r_1 = 0.1$ when using the binomial model. Alpha error is the solid red line to the right of the vertical dashed line; power is the red line to the left. Beta error is the dashed blue line to the left of the vertical dashed line; confidence is the blue line to the right.

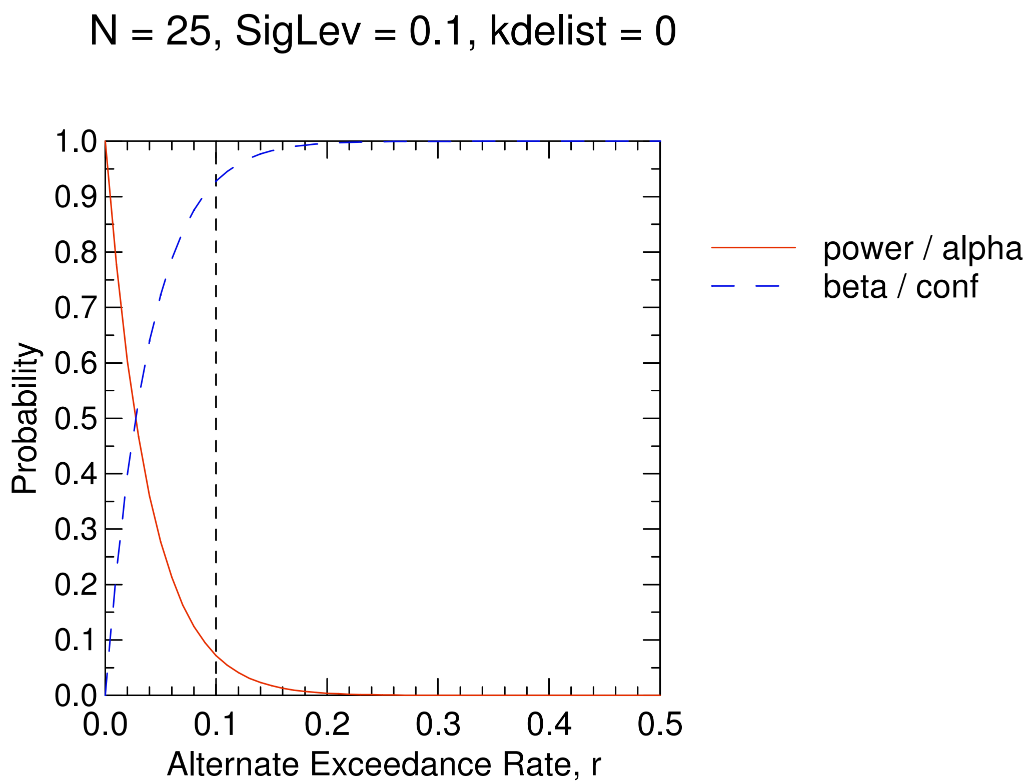
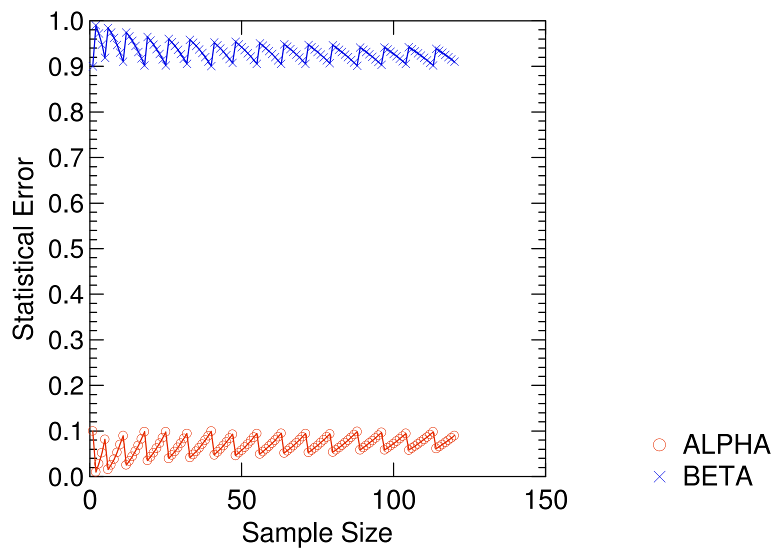


Figure 3. Statistical decision-making error rates for exceedance frequencies (Table 2) used in the draft SWRCB Policy (December 2, 2003 Version).

List when H_o: $r \leq 0.10$ is rejected



Delist when H_o: $r \geq 0.10$ is rejected

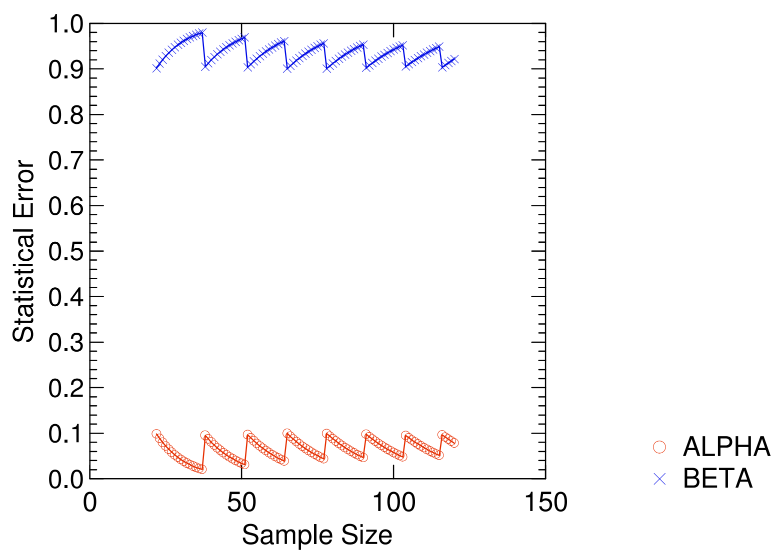


Figure 4. Magnification of the lower portion of Figure 1. Lowering the balanced error level (black vertical lines) increases the effect size (black horizontal lines). Three possible exceedance rate pair (r_1, r_2) realizations are shown.

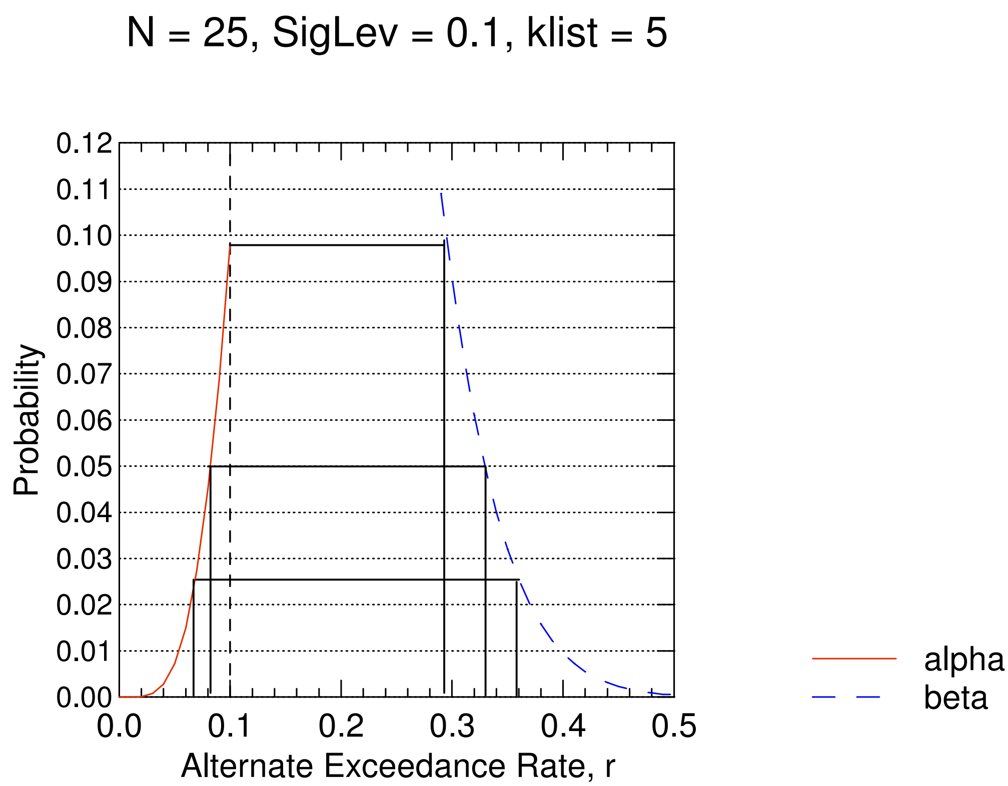


Figure 5. The balanced error approach for CWA 303(d) listing. Determination of *klist* = 5 by minimizing the absolute difference between alpha and beta errors. The minimized quantity is $|\alpha - \beta| = |\ 0.0980 - 0.2137\ | = 0.1157$. The beta to alpha ratio is 2.18.

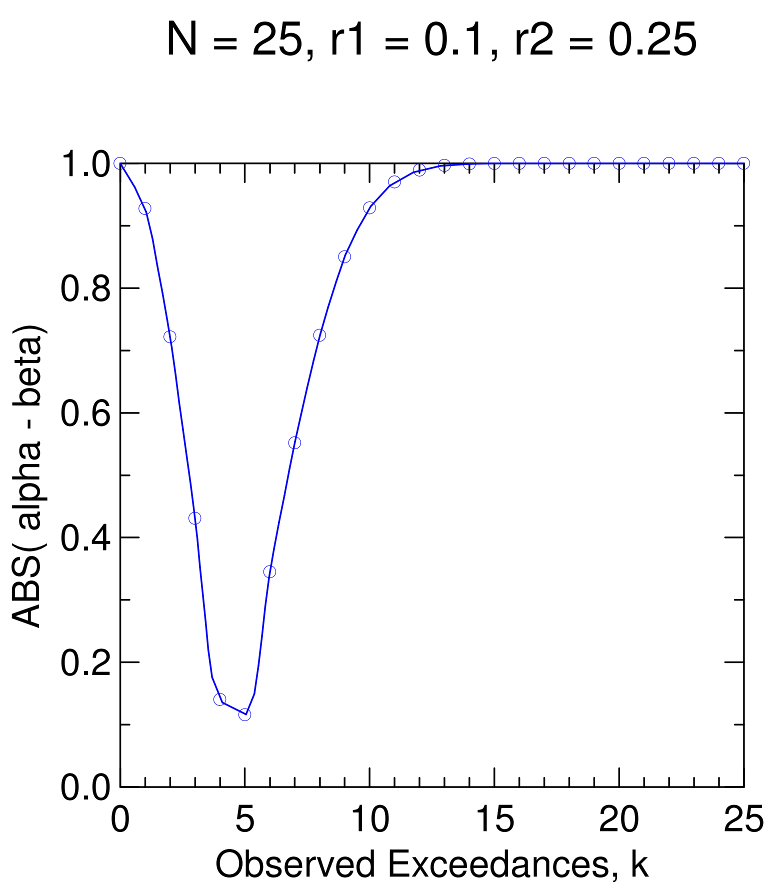
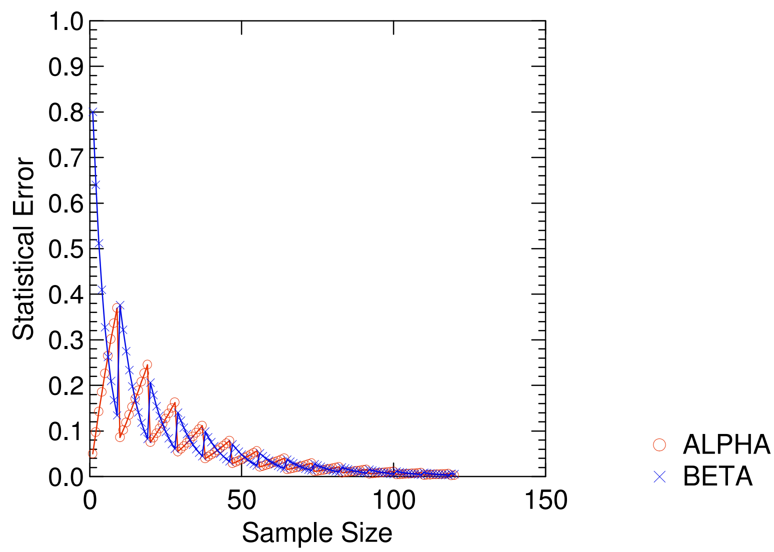


Figure 6. Balanced error rates associated with the sampling plan in Table 3. Effect size = 15%. Note the reversal of errors between listing and delisting.

List when H_o: $r \leq 0.05$ is rejected



Delist when H_o: $r \geq 0.20$ is rejected

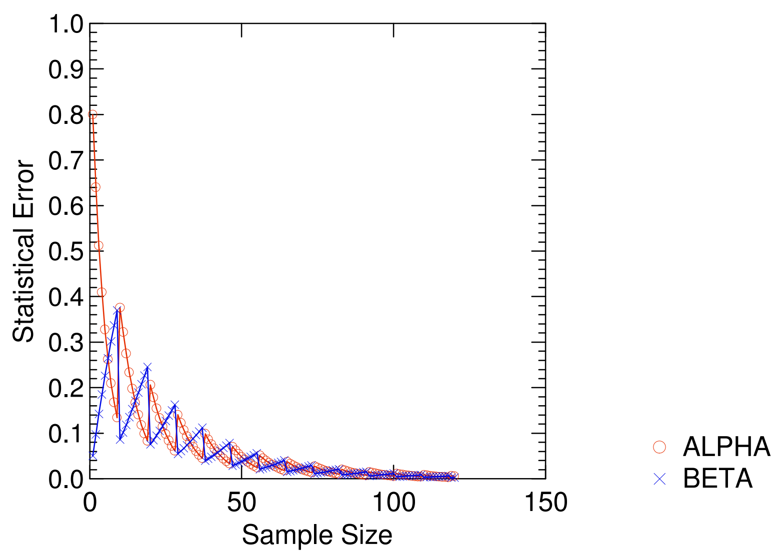
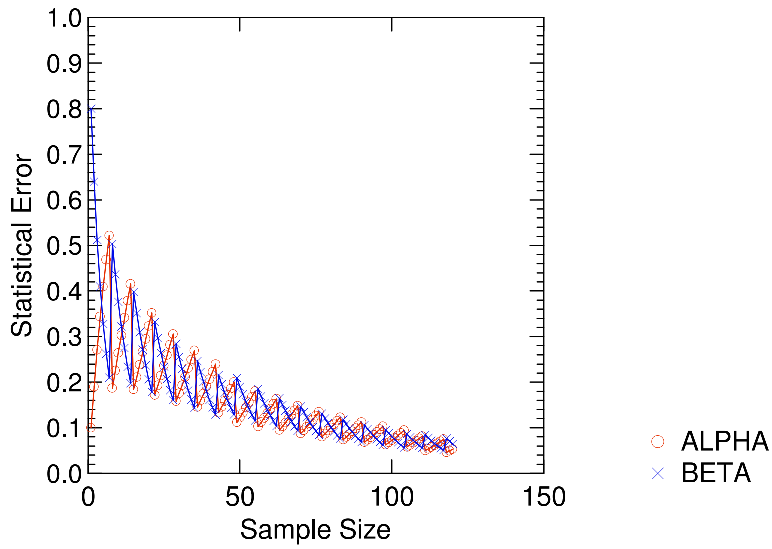


Figure 7. Balanced error rates associated with the sampling plan in Table 4. Effect size = 10%. Note the reversal of errors between listing and delisting.

List H_o: $r \leq 0.10$ is rejected



Delist when H_o: $r \geq 0.20$ is rejected

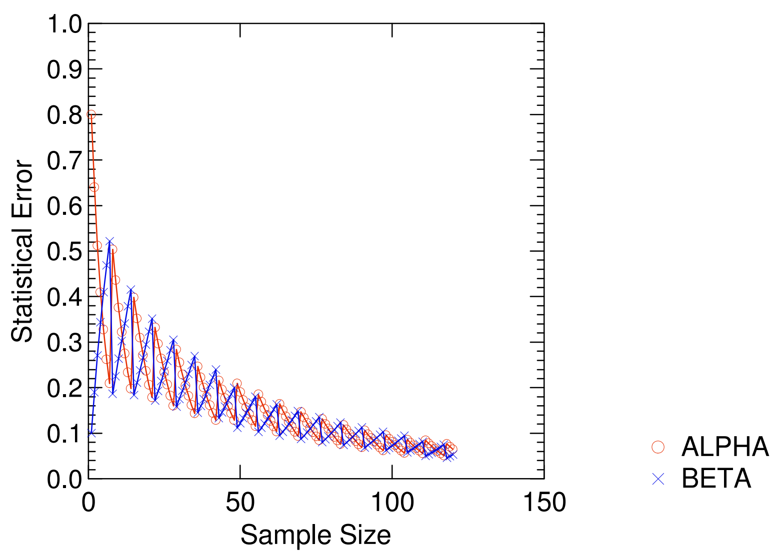
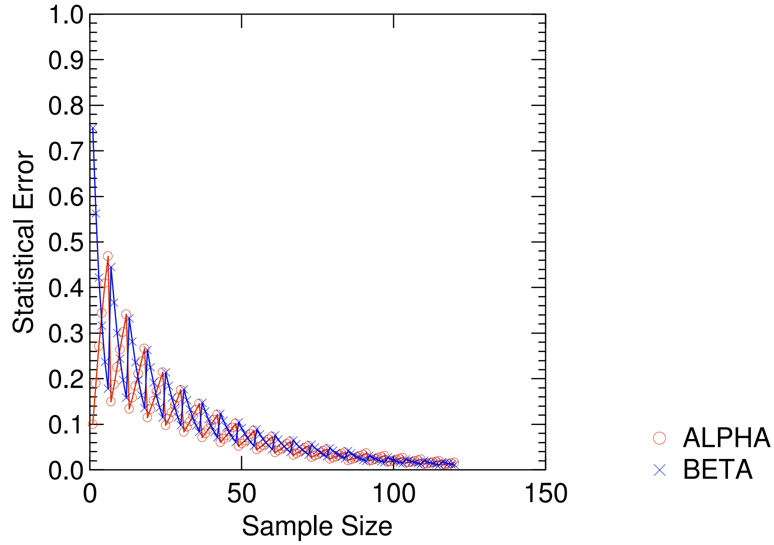


Figure 8. Balanced error rates associated with the sampling plan in Table 5. Effect size = 15%. Note the reversal of errors between listing and delisting.

List when H_o: $r \leq 0.10$ is rejected



Delist when H_o: $r \geq 0.25$ is rejected

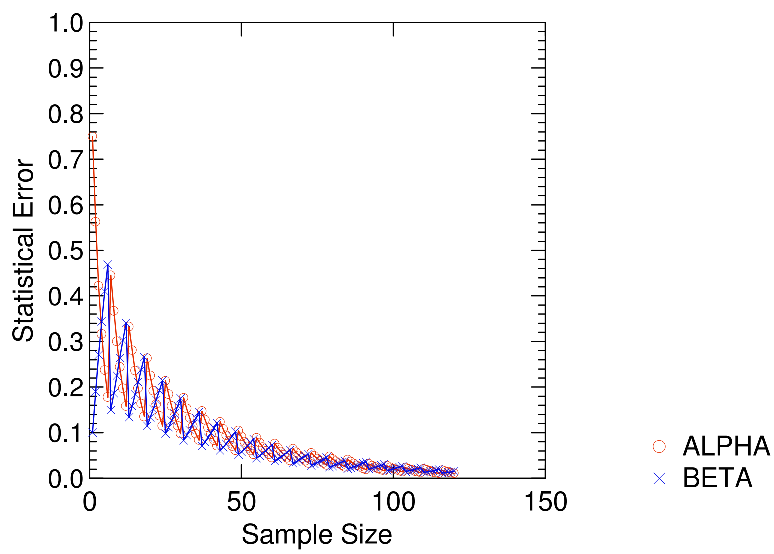
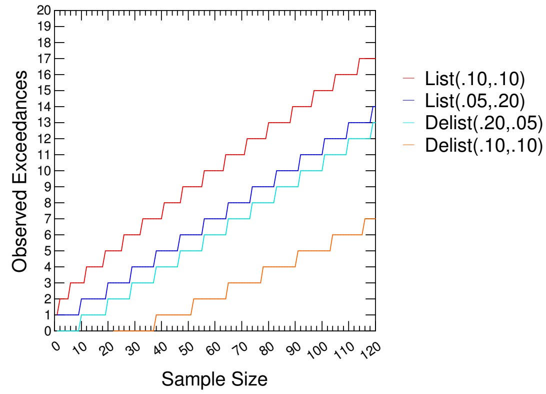
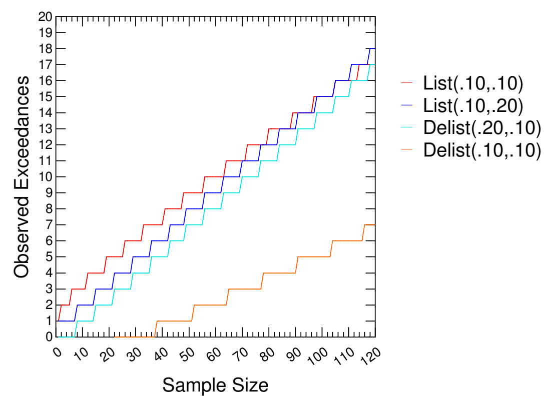


Figure 9. Comparison of Balanced Error Sampling Plans versus the Existing SWRCB Policy Sampling Plan of Table 2. Notation used is List(r_1, r_2) or Delist(r_1, r_2).

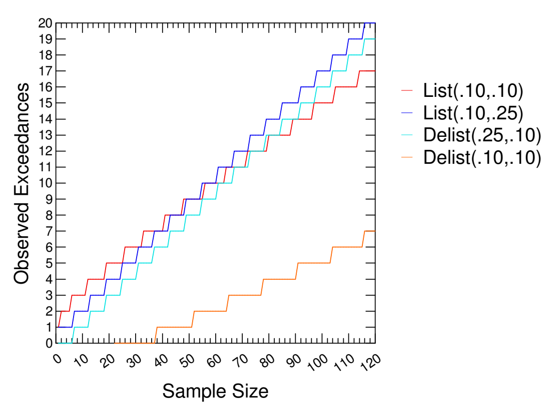
Policy vs.Table 3 Sampling Plan



Policy vs. Table 4 Sampling Plan



Policy vs. Table 5 Sampling Plan



Appendix. BinomBal.exe output used to make Tables 3-5.

```
===== Balanced Error Binomial Program Output =====   Apr 30, 2004  16:12:30

Null Hypothesis: r <= 0.05
Effect Size    : es = 0.15

 ssize       k     alpha      beta      min|a-b|      b/a
     1,      1,    0.0500,    0.8000,    0.7500,   16.0000
     2,      1,    0.0975,    0.6400,    0.5425,    6.5641
     3,      1,    0.1426,    0.5120,    0.3694,    3.5898
     4,      1,    0.1855,    0.4096,    0.2241,    2.2082
     5,      1,    0.2262,    0.3277,    0.1015,    1.4485
     6,      1,    0.2649,    0.2621,    0.0028,    0.9896
     7,      1,    0.3017,    0.2097,    0.0919,    0.6952
     8,      1,    0.3366,    0.1678,    0.1688,    0.4985
     9,      1,    0.3698,    0.1342,    0.2355,    0.3630
    10,      2,    0.0861,    0.3758,    0.2897,    4.3629
    11,      2,    0.1019,    0.3221,    0.2202,    3.1613
    12,      2,    0.1184,    0.2749,    0.1565,    2.3224
    13,      2,    0.1354,    0.2336,    0.0982,    1.7253
    14,      2,    0.1530,    0.1979,    0.0449,    1.2937
    15,      2,    0.1710,    0.1671,    0.0038,    0.9776
    16,      2,    0.1892,    0.1407,    0.0485,    0.7437
    17,      2,    0.2078,    0.1182,    0.0896,    0.5690
    18,      2,    0.2265,    0.0991,    0.1274,    0.4375
    19,      2,    0.2453,    0.0829,    0.1624,    0.3378
    20,      3,    0.0755,    0.2061,    0.1306,    2.7302
    21,      3,    0.0849,    0.1787,    0.0938,    2.1044
    22,      3,    0.0948,    0.1545,    0.0597,    1.6293
    23,      3,    0.1052,    0.1332,    0.0280,    1.2663
    24,      3,    0.1159,    0.1145,    0.0014,    0.9877
    25,      3,    0.1271,    0.0982,    0.0289,    0.7728
    26,      3,    0.1386,    0.0841,    0.0546,    0.6063
    27,      3,    0.1505,    0.0718,    0.0787,    0.4770
    28,      3,    0.1627,    0.0612,    0.1015,    0.3761
    29,      4,    0.0548,    0.1404,    0.0856,    2.5639
    30,      4,    0.0608,    0.1227,    0.0619,    2.0192
    31,      4,    0.0671,    0.1070,    0.0399,    1.5941
    32,      4,    0.0738,    0.0931,    0.0193,    1.2613
    33,      4,    0.0808,    0.0808,    0.0000,    1.0000
    34,      4,    0.0881,    0.0700,    0.0181,    0.7944
    35,      4,    0.0958,    0.0605,    0.0352,    0.6321
    36,      4,    0.1037,    0.0522,    0.0515,    0.5037
    37,      4,    0.1119,    0.0450,    0.0669,    0.4021
    38,      5,    0.0397,    0.0986,    0.0588,    2.4812
    39,      5,    0.0438,    0.0866,    0.0428,    1.9788
    40,      5,    0.0480,    0.0759,    0.0279,    1.5806
    41,      5,    0.0525,    0.0664,    0.0139,    1.2644
    42,      5,    0.0573,    0.0580,    0.0007,    1.0128
    43,      5,    0.0623,    0.0506,    0.0117,    0.8123
    44,      5,    0.0675,    0.0440,    0.0235,    0.6522
    45,      5,    0.0729,    0.0382,    0.0347,    0.5243
    46,      5,    0.0786,    0.0332,    0.0454,    0.4219
    47,      6,    0.0289,    0.0705,    0.0416,    2.4363
    48,      6,    0.0317,    0.0621,    0.0304,    1.9593
    49,      6,    0.0347,    0.0547,    0.0200,    1.5775
    50,      6,    0.0378,    0.0480,    0.0103,    1.2714
    51,      6,    0.0411,    0.0421,    0.0011,    1.0256
    52,      6,    0.0445,    0.0369,    0.0077,    0.8282
    53,      6,    0.0482,    0.0322,    0.0159,    0.6693
    54,      6,    0.0520,    0.0281,    0.0239,    0.5413
    55,      6,    0.0560,    0.0245,    0.0315,    0.4382
    56,      7,    0.0212,    0.0510,    0.0299,    2.4120
    57,      7,    0.0231,    0.0451,    0.0220,    1.9515
    58,      7,    0.0252,    0.0398,    0.0146,    1.5802
    59,      7,    0.0274,    0.0351,    0.0077,    1.2806
    60,      7,    0.0297,    0.0308,    0.0011,    1.0385
    61,      7,    0.0321,    0.0271,    0.0051,    0.8428
    62,      7,    0.0347,    0.0238,    0.0110,    0.6844
    63,      7,    0.0374,    0.0208,    0.0166,    0.5561
    64,      7,    0.0403,    0.0182,    0.0221,    0.4521
    65,      8,    0.0155,    0.0373,    0.0217,    2.4004
    66,      8,    0.0169,    0.0330,    0.0161,    1.9510
    67,      8,    0.0184,    0.0292,    0.0108,    1.5867
```

68,	8,	0.0200,	0.0258,	0.0058,	1.2912
69,	8,	0.0216,	0.0227,	0.0011,	1.0514
70,	8,	0.0234,	0.0200,	0.0034,	0.8566
71,	8,	0.0252,	0.0176,	0.0076,	0.6982
72,	8,	0.0272,	0.0155,	0.0117,	0.5694
73,	8,	0.0292,	0.0136,	0.0156,	0.4646
74,	9,	0.0114,	0.0274,	0.0160,	2.3971
75,	9,	0.0124,	0.0243,	0.0119,	1.9553
76,	9,	0.0135,	0.0215,	0.0080,	1.5957
77,	9,	0.0146,	0.0190,	0.0044,	1.3029
78,	9,	0.0158,	0.0168,	0.0010,	1.0643
79,	9,	0.0171,	0.0148,	0.0022,	0.8698
80,	9,	0.0184,	0.0131,	0.0053,	0.7112
81,	9,	0.0198,	0.0115,	0.0083,	0.5817
82,	9,	0.0213,	0.0101,	0.0112,	0.4760
83,	10,	0.0084,	0.0202,	0.0118,	2.3996
84,	10,	0.0092,	0.0180,	0.0088,	1.9630
85,	10,	0.0099,	0.0159,	0.0060,	1.6066
86,	10,	0.0107,	0.0141,	0.0034,	1.3153
87,	10,	0.0116,	0.0125,	0.0009,	1.0773
88,	10,	0.0125,	0.0111,	0.0015,	0.8827
89,	10,	0.0135,	0.0098,	0.0037,	0.7235
90,	10,	0.0145,	0.0086,	0.0059,	0.5932
91,	10,	0.0156,	0.0076,	0.0080,	0.4865
92,	11,	0.0062,	0.0150,	0.0088,	2.4065
93,	11,	0.0068,	0.0134,	0.0066,	1.9734
94,	11,	0.0073,	0.0119,	0.0045,	1.6188
95,	11,	0.0079,	0.0105,	0.0026,	1.3284
96,	11,	0.0086,	0.0093,	0.0008,	1.0904
97,	11,	0.0092,	0.0083,	0.0010,	0.8953
98,	11,	0.0099,	0.0073,	0.0026,	0.7354
99,	11,	0.0107,	0.0065,	0.0042,	0.6042
100,	11,	0.0115,	0.0057,	0.0058,	0.4965
101,	12,	0.0046,	0.0112,	0.0066,	2.4167
102,	12,	0.0050,	0.0100,	0.0049,	1.9858
103,	12,	0.0054,	0.0089,	0.0034,	1.6321
104,	12,	0.0059,	0.0079,	0.0020,	1.3419
105,	12,	0.0063,	0.0070,	0.0007,	1.1035
106,	12,	0.0068,	0.0062,	0.0006,	0.9078
107,	12,	0.0073,	0.0055,	0.0019,	0.7469
108,	12,	0.0079,	0.0048,	0.0030,	0.6147
109,	12,	0.0085,	0.0043,	0.0042,	0.5061
110,	13,	0.0034,	0.0084,	0.0049,	2.4295
111,	13,	0.0037,	0.0075,	0.0037,	1.9997
112,	13,	0.0040,	0.0066,	0.0026,	1.6464
113,	13,	0.0043,	0.0059,	0.0015,	1.3558
114,	13,	0.0047,	0.0052,	0.0005,	1.1168
115,	13,	0.0050,	0.0046,	0.0004,	0.9201
116,	13,	0.0054,	0.0041,	0.0013,	0.7583
117,	13,	0.0058,	0.0036,	0.0022,	0.6250
118,	13,	0.0063,	0.0032,	0.0030,	0.5153
119,	14,	0.0026,	0.0063,	0.0037,	2.4444
120,	14,	0.0028,	0.0056,	0.0028,	2.0150

===== Balanced Error Binomial Program Output =====   Apr 30, 2004 16:14:44

Null Hypothesis: r >= 0.2
Effect Size   : es = 0.15

ssize	k	alpha	beta	min|a-b|	b/a
1,	0,	0.8000,	0.0500,	0.7500,	0.0625
2,	0,	0.6400,	0.0975,	0.5425,	0.1523
3,	0,	0.5120,	0.1426,	0.3694,	0.2786
4,	0,	0.4096,	0.1855,	0.2241,	0.4529
5,	0,	0.3277,	0.2262,	0.1015,	0.6904
6,	0,	0.2621,	0.2649,	0.0028,	1.0105
7,	0,	0.2097,	0.3017,	0.0919,	1.4384
8,	0,	0.1678,	0.3366,	0.1688,	2.0062
9,	0,	0.1342,	0.3698,	0.2355,	2.7549
10,	1,	0.3758,	0.0861,	0.2897,	0.2292
11,	1,	0.3221,	0.1019,	0.2202,	0.3163
12,	1,	0.2749,	0.1184,	0.1565,	0.4306
13,	1,	0.2336,	0.1354,	0.0982,	0.5796
14,	1,	0.1979,	0.1530,	0.0449,	0.7730
15,	1,	0.1671,	0.1710,	0.0038,	1.0229

16,	1,	0.1407,	0.1892,	0.0485,	1.3446
17,	1,	0.1182,	0.2078,	0.0896,	1.7575
18,	1,	0.0991,	0.2265,	0.1274,	2.2858
19,	1,	0.0829,	0.2453,	0.1624,	2.9601
20,	2,	0.2061,	0.0755,	0.1306,	0.3663
21,	2,	0.1787,	0.0849,	0.0938,	0.4752
22,	2,	0.1545,	0.0948,	0.0597,	0.6138
23,	2,	0.1332,	0.1052,	0.0280,	0.7897
24,	2,	0.1145,	0.1159,	0.0014,	1.0125
25,	2,	0.0982,	0.1271,	0.0289,	1.2940
26,	2,	0.0841,	0.1386,	0.0546,	1.6492
27,	2,	0.0718,	0.1505,	0.0787,	2.0966
28,	2,	0.0612,	0.1627,	0.1015,	2.6592
29,	3,	0.1404,	0.0548,	0.0856,	0.3900
30,	3,	0.1227,	0.0608,	0.0619,	0.4952
31,	3,	0.1070,	0.0671,	0.0399,	0.6273
32,	3,	0.0931,	0.0738,	0.0193,	0.7928
33,	3,	0.0808,	0.0808,	0.0000,	1.0000
34,	3,	0.0700,	0.0881,	0.0181,	1.2589
35,	3,	0.0605,	0.0958,	0.0352,	1.5821
36,	3,	0.0522,	0.1037,	0.0515,	1.9851
37,	3,	0.0450,	0.1119,	0.0669,	2.4872
38,	4,	0.0986,	0.0397,	0.0588,	0.4030
39,	4,	0.0866,	0.0438,	0.0428,	0.5054
40,	4,	0.0759,	0.0480,	0.0279,	0.6327
41,	4,	0.0664,	0.0525,	0.0139,	0.7909
42,	4,	0.0580,	0.0573,	0.0007,	0.9874
43,	4,	0.0506,	0.0623,	0.0117,	1.2311
44,	4,	0.0440,	0.0675,	0.0235,	1.5332
45,	4,	0.0382,	0.0729,	0.0347,	1.9073
46,	4,	0.0332,	0.0786,	0.0454,	2.3703
47,	5,	0.0705,	0.0289,	0.0416,	0.4105
48,	5,	0.0621,	0.0317,	0.0304,	0.5104
49,	5,	0.0547,	0.0347,	0.0200,	0.6339
50,	5,	0.0480,	0.0378,	0.0103,	0.7866
51,	5,	0.0421,	0.0411,	0.0011,	0.9750
52,	5,	0.0369,	0.0445,	0.0077,	1.2075
53,	5,	0.0322,	0.0482,	0.0159,	1.4942
54,	5,	0.0281,	0.0520,	0.0239,	1.8474
55,	5,	0.0245,	0.0560,	0.0315,	2.2823
56,	6,	0.0510,	0.0212,	0.0299,	0.4146
57,	6,	0.0451,	0.0231,	0.0220,	0.5124
58,	6,	0.0398,	0.0252,	0.0146,	0.6328
59,	6,	0.0351,	0.0274,	0.0077,	0.7809
60,	6,	0.0308,	0.0297,	0.0011,	0.9629
61,	6,	0.0271,	0.0321,	0.0051,	1.1866
62,	6,	0.0238,	0.0347,	0.0110,	1.4612
63,	6,	0.0208,	0.0374,	0.0166,	1.7982
64,	6,	0.0182,	0.0403,	0.0221,	2.2117
65,	7,	0.0373,	0.0155,	0.0217,	0.4166
66,	7,	0.0330,	0.0169,	0.0161,	0.5126
67,	7,	0.0292,	0.0184,	0.0108,	0.6302
68,	7,	0.0258,	0.0200,	0.0058,	0.7745
69,	7,	0.0227,	0.0216,	0.0011,	0.9511
70,	7,	0.0200,	0.0234,	0.0034,	1.1675
71,	7,	0.0176,	0.0252,	0.0076,	1.4322
72,	7,	0.0155,	0.0272,	0.0117,	1.7562
73,	7,	0.0136,	0.0292,	0.0156,	2.1524
74,	8,	0.0274,	0.0114,	0.0160,	0.4172
75,	8,	0.0243,	0.0124,	0.0119,	0.5114
76,	8,	0.0215,	0.0135,	0.0080,	0.6267
77,	8,	0.0190,	0.0146,	0.0044,	0.7675
78,	8,	0.0168,	0.0158,	0.0010,	0.9396
79,	8,	0.0148,	0.0171,	0.0022,	1.1497
80,	8,	0.0131,	0.0184,	0.0053,	1.4062
81,	8,	0.0115,	0.0198,	0.0083,	1.7192
82,	8,	0.0101,	0.0213,	0.0112,	2.1010
83,	9,	0.0202,	0.0084,	0.0118,	0.4167
84,	9,	0.0180,	0.0092,	0.0088,	0.5094
85,	9,	0.0159,	0.0099,	0.0060,	0.6224
86,	9,	0.0141,	0.0107,	0.0034,	0.7603
87,	9,	0.0125,	0.0116,	0.0009,	0.9282
88,	9,	0.0111,	0.0125,	0.0015,	1.1329
89,	9,	0.0098,	0.0135,	0.0037,	1.3822
90,	9,	0.0086,	0.0145,	0.0059,	1.6858
91,	9,	0.0076,	0.0156,	0.0080,	2.0553

```
    92,      10,     0.0150,     0.0062,     0.0088,     0.4155
    93,      10,     0.0134,     0.0068,     0.0066,     0.5067
    94,      10,     0.0119,     0.0073,     0.0045,     0.6177
    95,      10,     0.0105,     0.0079,     0.0026,     0.7528
    96,      10,     0.0093,     0.0086,     0.0008,     0.9171
    97,      10,     0.0083,     0.0092,     0.0010,     1.1169
    98,      10,     0.0073,     0.0099,     0.0026,     1.3598
    99,      10,     0.0065,     0.0107,     0.0042,     1.6551
   100,      10,     0.0057,     0.0115,     0.0058,     2.0140
   101,      11,     0.0112,     0.0046,     0.0066,     0.4138
   102,      11,     0.0100,     0.0050,     0.0049,     0.5036
   103,      11,     0.0089,     0.0054,     0.0034,     0.6127
   104,      11,     0.0079,     0.0059,     0.0020,     0.7452
   105,      11,     0.0070,     0.0063,     0.0007,     0.9062
   106,      11,     0.0062,     0.0068,     0.0006,     1.1016
   107,      11,     0.0055,     0.0073,     0.0019,     1.3388
   108,      11,     0.0048,     0.0079,     0.0030,     1.6267
   109,      11,     0.0043,     0.0085,     0.0042,     1.9760
   110,      12,     0.0084,     0.0034,     0.0049,     0.4116
   111,      12,     0.0075,     0.0037,     0.0037,     0.5001
   112,      12,     0.0066,     0.0040,     0.0026,     0.6074
   113,      12,     0.0059,     0.0043,     0.0015,     0.7376
   114,      12,     0.0052,     0.0047,     0.0005,     0.8954
   115,      12,     0.0046,     0.0050,     0.0004,     1.0868
   116,      12,     0.0041,     0.0054,     0.0013,     1.3188
   117,      12,     0.0036,     0.0058,     0.0022,     1.6000
   118,      12,     0.0032,     0.0063,     0.0030,     1.9407
   119,      13,     0.0063,     0.0026,     0.0037,     0.4091
   120,      13,     0.0056,     0.0028,     0.0028,     0.4963
```

```
  ===== Balanced Error Binomial Program Output =====   Apr 30, 2004  16:15:58

Null Hypothesis: r <= 0.1
Effect Size    : es = 0.1

 ssize       k     alpha      beta      min|a-b|      b/a
     1,       1,     0.1000,     0.8000,     0.7000,     8.0000
     2,       1,     0.1900,     0.6400,     0.4500,     3.3684
     3,       1,     0.2710,     0.5120,     0.2410,     1.8893
     4,       1,     0.3439,     0.4096,     0.0657,     1.1910
     5,       1,     0.4095,     0.3277,     0.0818,     0.8002
     6,       1,     0.4686,     0.2621,     0.2064,     0.5595
     7,       1,     0.5217,     0.2097,     0.3120,     0.4020
     8,       2,     0.1869,     0.5033,     0.3164,     2.6930
     9,       2,     0.2252,     0.4362,     0.2110,     1.9373
    10,       2,     0.2639,     0.3758,     0.1119,     1.4241
    11,       2,     0.3026,     0.3221,     0.0195,     1.0644
    12,       2,     0.3410,     0.2749,     0.0661,     0.8061
    13,       2,     0.3787,     0.2336,     0.1450,     0.6170
    14,       2,     0.4154,     0.1979,     0.2175,     0.4765
    15,       3,     0.1841,     0.3980,     0.2140,     2.1625
    16,       3,     0.2108,     0.3518,     0.1411,     1.6695
    17,       3,     0.2382,     0.3096,     0.0714,     1.2998
    18,       3,     0.2662,     0.2713,     0.0051,     1.0193
    19,       3,     0.2946,     0.2369,     0.0577,     0.8042
    20,       3,     0.3231,     0.2061,     0.1170,     0.6379
    21,       3,     0.3516,     0.1787,     0.1729,     0.5083
    22,       4,     0.1719,     0.3320,     0.1601,     1.9313
    23,       4,     0.1927,     0.2965,     0.1038,     1.5386
    24,       4,     0.2143,     0.2639,     0.0496,     1.2315
    25,       4,     0.2364,     0.2340,     0.0024,     0.9898
    26,       4,     0.2591,     0.2068,     0.0522,     0.7984
    27,       4,     0.2821,     0.1823,     0.0998,     0.6462
    28,       4,     0.3054,     0.1602,     0.1453,     0.5244
    29,       5,     0.1584,     0.2839,     0.1255,     1.7921
    30,       5,     0.1755,     0.2552,     0.0797,     1.4544
    31,       5,     0.1932,     0.2287,     0.0355,     1.1839
    32,       5,     0.2115,     0.2044,     0.0071,     0.9664
    33,       5,     0.2303,     0.1821,     0.0482,     0.7908
    34,       5,     0.2496,     0.1619,     0.0877,     0.6485
    35,       5,     0.2693,     0.1435,     0.1258,     0.5329
    36,       6,     0.1454,     0.2464,     0.1010,     1.6945
    37,       6,     0.1598,     0.2225,     0.0627,     1.3924
    38,       6,     0.1747,     0.2004,     0.0256,     1.1467
    39,       6,     0.1903,     0.1800,     0.0102,     0.9461
```

40,	6,	0.2063,	0.1613,	0.0449,	0.7821
41,	6,	0.2227,	0.1442,	0.0785,	0.6476
42,	6,	0.2396,	0.1287,	0.1109,	0.5370
43,	7,	0.1333,	0.2158,	0.0826,	1.6196
44,	7,	0.1456,	0.1956,	0.0500,	1.3431
45,	7,	0.1585,	0.1768,	0.0183,	1.1155
46,	7,	0.1719,	0.1595,	0.0124,	0.9279
47,	7,	0.1857,	0.1436,	0.0422,	0.7729
48,	7,	0.2000,	0.1289,	0.0711,	0.6447
49,	8,	0.1119,	0.2091,	0.0973,	1.8694
50,	8,	0.1221,	0.1904,	0.0683,	1.5589
51,	8,	0.1329,	0.1730,	0.0401,	1.3017
52,	8,	0.1441,	0.1569,	0.0127,	1.0884
53,	8,	0.1558,	0.1420,	0.0139,	0.9111
54,	8,	0.1679,	0.1282,	0.0397,	0.7635
55,	8,	0.1804,	0.1156,	0.0649,	0.6405
56,	9,	0.1030,	0.1851,	0.0821,	1.7974
57,	9,	0.1120,	0.1689,	0.0568,	1.5075
58,	9,	0.1215,	0.1538,	0.0323,	1.2658
59,	9,	0.1314,	0.1398,	0.0084,	1.0640
60,	9,	0.1416,	0.1268,	0.0148,	0.8952
61,	9,	0.1523,	0.1148,	0.0375,	0.7539
62,	9,	0.1634,	0.1038,	0.0596,	0.6354
63,	10,	0.0948,	0.1645,	0.0697,	1.7358
64,	10,	0.1028,	0.1504,	0.0476,	1.4628
65,	10,	0.1112,	0.1372,	0.0260,	1.2339
66,	10,	0.1199,	0.1249,	0.0050,	1.0417
67,	10,	0.1290,	0.1136,	0.0155,	0.8801
68,	10,	0.1385,	0.1031,	0.0354,	0.7442
69,	10,	0.1484,	0.0934,	0.0549,	0.6297
70,	11,	0.0873,	0.1468,	0.0595,	1.6820
71,	11,	0.0944,	0.1343,	0.0399,	1.4231
72,	11,	0.1019,	0.1227,	0.0209,	1.2050
73,	11,	0.1097,	0.1120,	0.0023,	1.0210
74,	11,	0.1178,	0.1020,	0.0158,	0.8657
75,	11,	0.1263,	0.0928,	0.0335,	0.7345
76,	11,	0.1351,	0.0842,	0.0508,	0.6236
77,	12,	0.0804,	0.1313,	0.0510,	1.6341
78,	12,	0.0868,	0.1204,	0.0336,	1.3872
79,	12,	0.0934,	0.1101,	0.0167,	1.1784
80,	12,	0.1004,	0.1006,	0.0002,	1.0016
81,	12,	0.1077,	0.0918,	0.0160,	0.8518
82,	12,	0.1153,	0.0836,	0.0317,	0.7249
83,	12,	0.1232,	0.0760,	0.0472,	0.6171
84,	13,	0.0741,	0.1178,	0.0438,	1.5907
85,	13,	0.0798,	0.1081,	0.0283,	1.3543
86,	13,	0.0858,	0.0990,	0.0132,	1.1536
87,	13,	0.0921,	0.0906,	0.0015,	0.9832
88,	13,	0.0987,	0.0827,	0.0159,	0.8384
89,	13,	0.1055,	0.0755,	0.0300,	0.7153
90,	13,	0.1126,	0.0688,	0.0439,	0.6105
91,	14,	0.0683,	0.1059,	0.0376,	1.5510
92,	14,	0.0735,	0.0972,	0.0238,	1.3238
93,	14,	0.0789,	0.0892,	0.0103,	1.1304
94,	14,	0.0846,	0.0817,	0.0029,	0.9657
95,	14,	0.0905,	0.0747,	0.0158,	0.8254
96,	14,	0.0967,	0.0682,	0.0284,	0.7058
97,	14,	0.1031,	0.0622,	0.0408,	0.6037
98,	15,	0.0630,	0.0954,	0.0324,	1.5143
99,	15,	0.0677,	0.0877,	0.0200,	1.2953
100,	15,	0.0726,	0.0804,	0.0079,	1.1085
101,	15,	0.0777,	0.0737,	0.0040,	0.9490
102,	15,	0.0831,	0.0675,	0.0156,	0.8127
103,	15,	0.0887,	0.0617,	0.0269,	0.6963
104,	15,	0.0945,	0.0564,	0.0381,	0.5968
105,	16,	0.0581,	0.0860,	0.0279,	1.4801
106,	16,	0.0624,	0.0791,	0.0167,	1.2685
107,	16,	0.0668,	0.0727,	0.0059,	1.0876
108,	16,	0.0715,	0.0667,	0.0048,	0.9329
109,	16,	0.0763,	0.0611,	0.0152,	0.8004
110,	16,	0.0814,	0.0559,	0.0255,	0.6870
111,	17,	0.0500,	0.0844,	0.0344,	1.6871
112,	17,	0.0537,	0.0777,	0.0240,	1.4480
113,	17,	0.0575,	0.0715,	0.0140,	1.2432
114,	17,	0.0616,	0.0657,	0.0042,	1.0677
115,	17,	0.0658,	0.0604,	0.0054,	0.9173

116,	17,	0.0702,	0.0554,	0.0149,	0.7884
117,	17,	0.0748,	0.0507,	0.0241,	0.6778
118,	18,	0.0462,	0.0763,	0.0300,	1.6491
119,	18,	0.0496,	0.0703,	0.0207,	1.4177
120,	18,	0.0531,	0.0647,	0.0116,	1.2191

===== Balanced Error Binomial Program Output ===== Apr 30, 2004 16:17:13

Null Hypothesis: r >= 0.2
Effect Size : es = 0.1

ssize	k	alpha	beta	min|a-b|	b/a
1,	0,	0.8000,	0.1000,	0.7000,	0.1250
2,	0,	0.6400,	0.1900,	0.4500,	0.2969
3,	0,	0.5120,	0.2710,	0.2410,	0.5293
4,	0,	0.4096,	0.3439,	0.0657,	0.8396
5,	0,	0.3277,	0.4095,	0.0818,	1.2497
6,	0,	0.2621,	0.4686,	0.2064,	1.7874
7,	0,	0.2097,	0.5217,	0.3120,	2.4877
8,	1,	0.5033,	0.1869,	0.3164,	0.3713
9,	1,	0.4362,	0.2252,	0.2110,	0.5162
10,	1,	0.3758,	0.2639,	0.1119,	0.7022
11,	1,	0.3221,	0.3026,	0.0195,	0.9395
12,	1,	0.2749,	0.3410,	0.0661,	1.2405
13,	1,	0.2336,	0.3787,	0.1450,	1.6206
14,	1,	0.1979,	0.4154,	0.2175,	2.0988
15,	2,	0.3980,	0.1841,	0.2140,	0.4624
16,	2,	0.3518,	0.2108,	0.1411,	0.5990
17,	2,	0.3096,	0.2382,	0.0714,	0.7693
18,	2,	0.2713,	0.2662,	0.0051,	0.9811
19,	2,	0.2369,	0.2946,	0.0577,	1.2434
20,	2,	0.2061,	0.3231,	0.1170,	1.5677
21,	2,	0.1787,	0.3516,	0.1729,	1.9675
22,	3,	0.3320,	0.1719,	0.1601,	0.5178
23,	3,	0.2965,	0.1927,	0.1038,	0.6500
24,	3,	0.2639,	0.2143,	0.0496,	0.8120
25,	3,	0.2340,	0.2364,	0.0024,	1.0103
26,	3,	0.2068,	0.2591,	0.0522,	1.2525
27,	3,	0.1823,	0.2821,	0.0998,	1.5476
28,	3,	0.1602,	0.3054,	0.1453,	1.9068
29,	4,	0.2839,	0.1584,	0.1255,	0.5580
30,	4,	0.2552,	0.1755,	0.0797,	0.6876
31,	4,	0.2287,	0.1932,	0.0355,	0.8447
32,	4,	0.2044,	0.2115,	0.0071,	1.0348
33,	4,	0.1821,	0.2303,	0.0482,	1.2646
34,	4,	0.1619,	0.2496,	0.0877,	1.5420
35,	4,	0.1435,	0.2693,	0.1258,	1.8764
36,	5,	0.2464,	0.1454,	0.1010,	0.5902
37,	5,	0.2225,	0.1598,	0.0627,	0.7182
38,	5,	0.2004,	0.1747,	0.0256,	0.8721
39,	5,	0.1800,	0.1903,	0.0102,	1.0569
40,	5,	0.1613,	0.2063,	0.0449,	1.2786
41,	5,	0.1442,	0.2227,	0.0785,	1.5442
42,	5,	0.1287,	0.2396,	0.1109,	1.8622
43,	6,	0.2158,	0.1333,	0.0826,	0.6174
44,	6,	0.1956,	0.1456,	0.0500,	0.7446
45,	6,	0.1768,	0.1585,	0.0183,	0.8964
46,	6,	0.1595,	0.1719,	0.0124,	1.0777
47,	6,	0.1436,	0.1857,	0.0422,	1.2938
48,	6,	0.1289,	0.2000,	0.0711,	1.5512
49,	7,	0.2091,	0.1119,	0.0973,	0.5349
50,	7,	0.1904,	0.1221,	0.0683,	0.6415
51,	7,	0.1730,	0.1329,	0.0401,	0.7682
52,	7,	0.1569,	0.1441,	0.0127,	0.9188
53,	7,	0.1420,	0.1558,	0.0139,	1.0976
54,	7,	0.1282,	0.1679,	0.0397,	1.3098
55,	7,	0.1156,	0.1804,	0.0649,	1.5614
56,	8,	0.1851,	0.1030,	0.0821,	0.5564
57,	8,	0.1689,	0.1120,	0.0568,	0.6633
58,	8,	0.1538,	0.1215,	0.0323,	0.7900
59,	8,	0.1398,	0.1314,	0.0084,	0.9399
60,	8,	0.1268,	0.1416,	0.0148,	1.1171
61,	8,	0.1148,	0.1523,	0.0375,	1.3265
62,	8,	0.1038,	0.1634,	0.0596,	1.5739
63,	9,	0.1645,	0.0948,	0.0697,	0.5761

64,	9,	0.1504,	0.1028,	0.0476,	0.6836
65,	9,	0.1372,	0.1112,	0.0260,	0.8104
66,	9,	0.1249,	0.1199,	0.0050,	0.9600
67,	9,	0.1136,	0.1290,	0.0155,	1.1362
68,	9,	0.1031,	0.1385,	0.0354,	1.3437
69,	9,	0.0934,	0.1484,	0.0549,	1.5881
70,	10,	0.1468,	0.0873,	0.0595,	0.5945
71,	10,	0.1343,	0.0944,	0.0399,	0.7027
72,	10,	0.1227,	0.1019,	0.0209,	0.8299
73,	10,	0.1120,	0.1097,	0.0023,	0.9794
74,	10,	0.1020,	0.1178,	0.0158,	1.1551
75,	10,	0.0928,	0.1263,	0.0335,	1.3614
76,	10,	0.0842,	0.1351,	0.0508,	1.6037
77,	11,	0.1313,	0.0804,	0.0510,	0.6120
78,	11,	0.1204,	0.0868,	0.0336,	0.7209
79,	11,	0.1101,	0.0934,	0.0167,	0.8486
80,	11,	0.1006,	0.1004,	0.0002,	0.9984
81,	11,	0.0918,	0.1077,	0.0160,	1.1739
82,	11,	0.0836,	0.1153,	0.0317,	1.3796
83,	11,	0.0760,	0.1232,	0.0472,	1.6204
84,	12,	0.1178,	0.0741,	0.0438,	0.6287
85,	12,	0.1081,	0.0798,	0.0283,	0.7384
86,	12,	0.0990,	0.0858,	0.0132,	0.8668
87,	12,	0.0906,	0.0921,	0.0015,	1.0171
88,	12,	0.0827,	0.0987,	0.0159,	1.1927
89,	12,	0.0755,	0.1055,	0.0300,	1.3981
90,	12,	0.0688,	0.1126,	0.0439,	1.6380
91,	13,	0.1059,	0.0683,	0.0376,	0.6447
92,	13,	0.0972,	0.0735,	0.0238,	0.7554
93,	13,	0.0892,	0.0789,	0.0103,	0.8847
94,	13,	0.0817,	0.0846,	0.0029,	1.0355
95,	13,	0.0747,	0.0905,	0.0158,	1.2115
96,	13,	0.0682,	0.0967,	0.0284,	1.4169
97,	13,	0.0622,	0.1031,	0.0408,	1.6564
98,	14,	0.0954,	0.0630,	0.0324,	0.6604
99,	14,	0.0877,	0.0677,	0.0200,	0.7720
100,	14,	0.0804,	0.0726,	0.0079,	0.9022
101,	14,	0.0737,	0.0777,	0.0040,	1.0538
102,	14,	0.0675,	0.0831,	0.0156,	1.2304
103,	14,	0.0617,	0.0887,	0.0269,	1.4361
104,	14,	0.0564,	0.0945,	0.0381,	1.6756
105,	15,	0.0860,	0.0581,	0.0279,	0.6756
106,	15,	0.0791,	0.0624,	0.0167,	0.7883
107,	15,	0.0727,	0.0668,	0.0059,	0.9194
108,	15,	0.0667,	0.0715,	0.0048,	1.0720
109,	15,	0.0611,	0.0763,	0.0152,	1.2494
110,	15,	0.0559,	0.0814,	0.0255,	1.4556
111,	16,	0.0844,	0.0500,	0.0344,	0.5927
112,	16,	0.0777,	0.0537,	0.0240,	0.6906
113,	16,	0.0715,	0.0575,	0.0140,	0.8044
114,	16,	0.0657,	0.0616,	0.0042,	0.9366
115,	16,	0.0604,	0.0658,	0.0054,	1.0901
116,	16,	0.0554,	0.0702,	0.0149,	1.2684
117,	16,	0.0507,	0.0748,	0.0241,	1.4754
118,	17,	0.0763,	0.0462,	0.0300,	0.6064
119,	17,	0.0703,	0.0496,	0.0207,	0.7054
120,	17,	0.0647,	0.0531,	0.0116,	0.8203

===== Balanced Error Binomial Program Output =====   May 10, 2004 11:02:56

Null Hypothesis: r <= 0.10
Effect Size   : es = 0.15

ssize	k	alpha	beta	min|a-b|	b/a
1,	1,	0.1000,	0.7500,	0.6500,	7.5000
2,	1,	0.1900,	0.5625,	0.3725,	2.9605
3,	1,	0.2710,	0.4219,	0.1509,	1.5567
4,	1,	0.3439,	0.3164,	0.0275,	0.9201
5,	1,	0.4095,	0.2373,	0.1722,	0.5795
6,	1,	0.4686,	0.1780,	0.2906,	0.3798
7,	2,	0.1497,	0.4449,	0.2953,	2.9724
8,	2,	0.1869,	0.3671,	0.1802,	1.9641
9,	2,	0.2252,	0.3003,	0.0752,	1.3339
10,	2,	0.2639,	0.2440,	0.0199,	0.9247
11,	2,	0.3026,	0.1971,	0.1055,	0.6513

12,	2,	0.3410,	0.1584,	0.1826,	0.4645
13,	3,	0.1339,	0.3326,	0.1987,	2.4843
14,	3,	0.1584,	0.2811,	0.1228,	1.7752
15,	3,	0.1841,	0.2361,	0.0520,	1.2827
16,	3,	0.2108,	0.1971,	0.0136,	0.9353
17,	3,	0.2382,	0.1637,	0.0745,	0.6872
18,	3,	0.2662,	0.1353,	0.1309,	0.5083
19,	4,	0.1150,	0.2631,	0.1481,	2.2878
20,	4,	0.1330,	0.2252,	0.0922,	1.6935
21,	4,	0.1520,	0.1917,	0.0397,	1.2614
22,	4,	0.1719,	0.1624,	0.0095,	0.9445
23,	4,	0.1927,	0.1370,	0.0558,	0.7106
24,	4,	0.2143,	0.1150,	0.0992,	0.5368
25,	5,	0.0980,	0.2137,	0.1157,	2.1812
26,	5,	0.1118,	0.1844,	0.0725,	1.6485
27,	5,	0.1266,	0.1583,	0.0318,	1.2509
28,	5,	0.1421,	0.1354,	0.0067,	0.9527
29,	5,	0.1584,	0.1153,	0.0431,	0.7279
30,	5,	0.1755,	0.0979,	0.0776,	0.5577
31,	6,	0.0834,	0.1764,	0.0930,	2.1148
32,	6,	0.0944,	0.1530,	0.0586,	1.6208
33,	6,	0.1061,	0.1322,	0.0261,	1.2458
34,	6,	0.1185,	0.1138,	0.0047,	0.9601
35,	6,	0.1316,	0.0976,	0.0340,	0.7416
36,	6,	0.1454,	0.0835,	0.0619,	0.5741
37,	7,	0.0711,	0.1472,	0.0761,	2.0702
38,	7,	0.0800,	0.1282,	0.0482,	1.6028
39,	7,	0.0894,	0.1112,	0.0218,	1.2437
40,	7,	0.0995,	0.0962,	0.0033,	0.9669
41,	7,	0.1102,	0.0830,	0.0272,	0.7531
42,	7,	0.1214,	0.0714,	0.0501,	0.5876
43,	8,	0.0607,	0.1237,	0.0630,	2.0388
44,	8,	0.0679,	0.1081,	0.0401,	1.5908
45,	8,	0.0757,	0.0941,	0.0184,	1.2434
46,	8,	0.0840,	0.0817,	0.0022,	0.9734
47,	8,	0.0928,	0.0708,	0.0220,	0.7631
48,	8,	0.1021,	0.0611,	0.0409,	0.5991
49,	9,	0.0519,	0.1046,	0.0527,	2.0161
50,	9,	0.0579,	0.0916,	0.0337,	1.5829
51,	9,	0.0643,	0.0800,	0.0157,	1.2445
52,	9,	0.0712,	0.0697,	0.0014,	0.9796
53,	9,	0.0785,	0.0606,	0.0179,	0.7721
54,	9,	0.0862,	0.0525,	0.0337,	0.6092
55,	10,	0.0444,	0.0888,	0.0444,	1.9995
56,	10,	0.0494,	0.0780,	0.0285,	1.5778
57,	10,	0.0548,	0.0683,	0.0135,	1.2464
58,	10,	0.0605,	0.0596,	0.0009,	0.9856
59,	10,	0.0666,	0.0520,	0.0146,	0.7802
60,	10,	0.0731,	0.0452,	0.0279,	0.6182
61,	11,	0.0381,	0.0757,	0.0376,	1.9873
62,	11,	0.0423,	0.0666,	0.0243,	1.5747
63,	11,	0.0468,	0.0584,	0.0116,	1.2490
64,	11,	0.0516,	0.0511,	0.0004,	0.9915
65,	11,	0.0567,	0.0447,	0.0120,	0.7878
66,	11,	0.0621,	0.0389,	0.0232,	0.6264
67,	12,	0.0327,	0.0647,	0.0320,	1.9784
68,	12,	0.0362,	0.0570,	0.0208,	1.5733
69,	12,	0.0400,	0.0501,	0.0101,	1.2521
70,	12,	0.0441,	0.0439,	0.0001,	0.9972
71,	12,	0.0484,	0.0385,	0.0099,	0.7948
72,	12,	0.0530,	0.0336,	0.0194,	0.6339
73,	13,	0.0281,	0.0555,	0.0274,	1.9721
74,	13,	0.0311,	0.0489,	0.0178,	1.5730
75,	13,	0.0343,	0.0431,	0.0088,	1.2556
76,	13,	0.0377,	0.0378,	0.0001,	1.0029
77,	13,	0.0414,	0.0332,	0.0082,	0.8015
78,	13,	0.0453,	0.0290,	0.0163,	0.6410
79,	14,	0.0242,	0.0477,	0.0234,	1.9678
80,	14,	0.0267,	0.0421,	0.0153,	1.5738
81,	14,	0.0295,	0.0371,	0.0076,	1.2594
82,	14,	0.0324,	0.0326,	0.0003,	1.0084
83,	14,	0.0355,	0.0287,	0.0068,	0.8079
84,	14,	0.0388,	0.0251,	0.0137,	0.6476
85,	15,	0.0209,	0.0410,	0.0202,	1.9652
86,	15,	0.0230,	0.0363,	0.0132,	1.5753
87,	15,	0.0253,	0.0320,	0.0067,	1.2635

88,	15,	0.0278,	0.0282,	0.0004,	1.0140
89,	15,	0.0304,	0.0248,	0.0057,	0.8141
90,	15,	0.0333,	0.0218,	0.0115,	0.6539
91,	15,	0.0363,	0.0191,	0.0172,	0.5255
92,	16,	0.0198,	0.0313,	0.0115,	1.5776
93,	16,	0.0218,	0.0276,	0.0058,	1.2678
94,	16,	0.0239,	0.0244,	0.0005,	1.0194
95,	16,	0.0262,	0.0215,	0.0047,	0.8200
96,	16,	0.0286,	0.0189,	0.0097,	0.6599
97,	16,	0.0312,	0.0166,	0.0146,	0.5313
98,	17,	0.0171,	0.0271,	0.0099,	1.5803
99,	17,	0.0188,	0.0239,	0.0051,	1.2724
100,	17,	0.0206,	0.0211,	0.0005,	1.0248
101,	17,	0.0225,	0.0186,	0.0039,	0.8258
102,	17,	0.0246,	0.0164,	0.0082,	0.6657
103,	17,	0.0268,	0.0144,	0.0124,	0.5368
104,	18,	0.0148,	0.0234,	0.0086,	1.5836
105,	18,	0.0162,	0.0207,	0.0045,	1.2771
106,	18,	0.0178,	0.0183,	0.0005,	1.0302
107,	18,	0.0194,	0.0161,	0.0033,	0.8314
108,	18,	0.0212,	0.0142,	0.0070,	0.6712
109,	18,	0.0231,	0.0125,	0.0106,	0.5421
110,	19,	0.0128,	0.0203,	0.0075,	1.5872
111,	19,	0.0140,	0.0180,	0.0039,	1.2819
112,	19,	0.0153,	0.0159,	0.0005,	1.0356
113,	19,	0.0167,	0.0140,	0.0027,	0.8370
114,	19,	0.0183,	0.0124,	0.0059,	0.6766
115,	19,	0.0199,	0.0109,	0.0090,	0.5472
116,	20,	0.0111,	0.0176,	0.0065,	1.5912
117,	20,	0.0121,	0.0156,	0.0035,	1.2868
118,	20,	0.0132,	0.0138,	0.0005,	1.0410
119,	20,	0.0145,	0.0122,	0.0023,	0.8424
120,	20,	0.0158,	0.0108,	0.0050,	0.6818

===== Balanced Error Binomial Program Output ===== May 10, 2004 11:07:10

Null Hypothesis: r >= 0.25
Effect Size : es = 0.15

ssize	k	alpha	beta	min|a-b|	b/a
1,	0,	0.7500,	0.1000,	0.6500,	0.1333
2,	0,	0.5625,	0.1900,	0.3725,	0.3378
3,	0,	0.4219,	0.2710,	0.1509,	0.6424
4,	0,	0.3164,	0.3439,	0.0275,	1.0869
5,	0,	0.2373,	0.4095,	0.1722,	1.7257
6,	0,	0.1780,	0.4686,	0.2906,	2.6327
7,	1,	0.4449,	0.1497,	0.2953,	0.3364
8,	1,	0.3671,	0.1869,	0.1802,	0.5091
9,	1,	0.3003,	0.2252,	0.0752,	0.7497
10,	1,	0.2440,	0.2639,	0.0199,	1.0814
11,	1,	0.1971,	0.3026,	0.1055,	1.5355
12,	1,	0.1584,	0.3410,	0.1826,	2.1530
13,	2,	0.3326,	0.1339,	0.1987,	0.4025
14,	2,	0.2811,	0.1584,	0.1228,	0.5633
15,	2,	0.2361,	0.1841,	0.0520,	0.7796
16,	2,	0.1971,	0.2108,	0.0136,	1.0692
17,	2,	0.1637,	0.2382,	0.0745,	1.4551
18,	2,	0.1353,	0.2662,	0.1309,	1.9674
19,	3,	0.2631,	0.1150,	0.1481,	0.4371
20,	3,	0.2252,	0.1330,	0.0922,	0.5905
21,	3,	0.1917,	0.1520,	0.0397,	0.7928
22,	3,	0.1624,	0.1719,	0.0095,	1.0587
23,	3,	0.1370,	0.1927,	0.0558,	1.4072
24,	3,	0.1150,	0.2143,	0.0992,	1.8629
25,	4,	0.2137,	0.0980,	0.1157,	0.4585
26,	4,	0.1844,	0.1118,	0.0725,	0.6066
27,	4,	0.1583,	0.1266,	0.0318,	0.7994
28,	4,	0.1354,	0.1421,	0.0067,	1.0497
29,	4,	0.1153,	0.1584,	0.0431,	1.3739
30,	4,	0.0979,	0.1755,	0.0776,	1.7932
31,	5,	0.1764,	0.0834,	0.0930,	0.4729
32,	5,	0.1530,	0.0944,	0.0586,	0.6170
33,	5,	0.1322,	0.1061,	0.0261,	0.8027
34,	5,	0.1138,	0.1185,	0.0047,	1.0416
35,	5,	0.0976,	0.1316,	0.0340,	1.3484

36,	5,	0.0835,	0.1454,	0.0619,	1.7420
37,	6,	0.1472,	0.0711,	0.0761,	0.4831
38,	6,	0.1282,	0.0800,	0.0482,	0.6239
39,	6,	0.1112,	0.0894,	0.0218,	0.8041
40,	6,	0.0962,	0.0995,	0.0033,	1.0342
41,	6,	0.0830,	0.1102,	0.0272,	1.3278
42,	6,	0.0714,	0.1214,	0.0501,	1.7020
43,	7,	0.1237,	0.0607,	0.0630,	0.4905
44,	7,	0.1081,	0.0679,	0.0401,	0.6286
45,	7,	0.0941,	0.0757,	0.0184,	0.8042
46,	7,	0.0817,	0.0840,	0.0022,	1.0273
47,	7,	0.0708,	0.0928,	0.0220,	1.3104
48,	7,	0.0611,	0.1021,	0.0409,	1.6693
49,	8,	0.1046,	0.0519,	0.0527,	0.4960
50,	8,	0.0916,	0.0579,	0.0337,	0.6318
51,	8,	0.0800,	0.0643,	0.0157,	0.8036
52,	8,	0.0697,	0.0712,	0.0014,	1.0208
53,	8,	0.0606,	0.0785,	0.0179,	1.2952
54,	8,	0.0525,	0.0862,	0.0337,	1.6416
55,	9,	0.0888,	0.0444,	0.0444,	0.5001
56,	9,	0.0780,	0.0494,	0.0285,	0.6338
57,	9,	0.0683,	0.0548,	0.0135,	0.8023
58,	9,	0.0596,	0.0605,	0.0009,	1.0146
59,	9,	0.0520,	0.0666,	0.0146,	1.2817
60,	9,	0.0452,	0.0731,	0.0279,	1.6177
61,	10,	0.0757,	0.0381,	0.0376,	0.5032
62,	10,	0.0666,	0.0423,	0.0243,	0.6350
63,	10,	0.0584,	0.0468,	0.0116,	0.8007
64,	10,	0.0511,	0.0516,	0.0004,	1.0086
65,	10,	0.0447,	0.0567,	0.0120,	1.2694
66,	10,	0.0389,	0.0621,	0.0232,	1.5965
67,	11,	0.0647,	0.0327,	0.0320,	0.5054
68,	11,	0.0570,	0.0362,	0.0208,	0.6356
69,	11,	0.0501,	0.0400,	0.0101,	0.7987
70,	11,	0.0439,	0.0441,	0.0001,	1.0028
71,	11,	0.0385,	0.0484,	0.0099,	1.2581
72,	11,	0.0336,	0.0530,	0.0194,	1.5774
73,	12,	0.0555,	0.0281,	0.0274,	0.5071
74,	12,	0.0489,	0.0311,	0.0178,	0.6357
75,	12,	0.0431,	0.0343,	0.0088,	0.7964
76,	12,	0.0378,	0.0377,	0.0001,	0.9971
77,	12,	0.0332,	0.0414,	0.0082,	1.2476
78,	12,	0.0290,	0.0453,	0.0163,	1.5601
79,	13,	0.0477,	0.0242,	0.0234,	0.5082
80,	13,	0.0421,	0.0267,	0.0153,	0.6354
81,	13,	0.0371,	0.0295,	0.0076,	0.7940
82,	13,	0.0326,	0.0324,	0.0003,	0.9916
83,	13,	0.0287,	0.0355,	0.0068,	1.2377
84,	13,	0.0251,	0.0388,	0.0137,	1.5441
85,	14,	0.0410,	0.0209,	0.0202,	0.5089
86,	14,	0.0363,	0.0230,	0.0132,	0.6348
87,	14,	0.0320,	0.0253,	0.0067,	0.7914
88,	14,	0.0282,	0.0278,	0.0004,	0.9862
89,	14,	0.0248,	0.0304,	0.0057,	1.2284
90,	14,	0.0218,	0.0333,	0.0115,	1.5292
91,	14,	0.0191,	0.0363,	0.0172,	1.9030
92,	15,	0.0313,	0.0198,	0.0115,	0.6339
93,	15,	0.0276,	0.0218,	0.0058,	0.7887
94,	15,	0.0244,	0.0239,	0.0005,	0.9810
95,	15,	0.0215,	0.0262,	0.0047,	1.2195
96,	15,	0.0189,	0.0286,	0.0097,	1.5153
97,	15,	0.0166,	0.0312,	0.0146,	1.8822
98,	16,	0.0271,	0.0171,	0.0099,	0.6328
99,	16,	0.0239,	0.0188,	0.0051,	0.7859
100,	16,	0.0211,	0.0206,	0.0005,	0.9758
101,	16,	0.0186,	0.0225,	0.0039,	1.2109
102,	16,	0.0164,	0.0246,	0.0082,	1.5022
103,	16,	0.0144,	0.0268,	0.0124,	1.8629
104,	17,	0.0234,	0.0148,	0.0086,	0.6315
105,	17,	0.0207,	0.0162,	0.0045,	0.7831
106,	17,	0.0183,	0.0178,	0.0005,	0.9706
107,	17,	0.0161,	0.0194,	0.0033,	1.2027
108,	17,	0.0142,	0.0212,	0.0070,	1.4898
109,	17,	0.0125,	0.0231,	0.0106,	1.8448
110,	18,	0.0203,	0.0128,	0.0075,	0.6300
111,	18,	0.0180,	0.0140,	0.0039,	0.7801

112,	18,	0.0159,	0.0153,	0.0005,	0.9656
113,	18,	0.0140,	0.0167,	0.0027,	1.1948
114,	18,	0.0124,	0.0183,	0.0059,	1.4779
115,	18,	0.0109,	0.0199,	0.0090,	1.8276
116,	19,	0.0176,	0.0111,	0.0065,	0.6284
117,	19,	0.0156,	0.0121,	0.0035,	0.7771
118,	19,	0.0138,	0.0132,	0.0005,	0.9606
119,	19,	0.0122,	0.0145,	0.0023,	1.1871
120,	19,	0.0108,	0.0158,	0.0050,	1.4666